

To Mr. Goss of the Weather

THIS DOCUMENT AND EACH AND EVERY  
PAGE HEREIN IS HEREBY RECLASSIFIED  
FROM Conf TO Unclass.  
AS PER LETTER DATED 11/26/36 RECLASS.  
Notice # 122

Source of Acquisition  
CASI Acquired

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

CHANCE VOUGHT CORPORATION LIBRARY

Special Report # 55

RELATIVE EFFICIENCIES AND DESIGN CHARTS FOR  
VARIOUS ENGINE-PROPELLER COMBINATIONS

By David Biermann  
Langley Memorial Aeronautical Laboratory

SPECIAL RPT-55

September 1936

CHANCE VOUGHT CORPORATION LIBRARY

RELATIVE EFFICIENCIES AND DESIGN CHARTS FOR  
VARIOUS ENGINE-PROPELLER COMBINATIONS

By David Biermann

SUMMARY

The relative efficiencies of various engine-propeller combinations were the subject of a study that covered the important flight conditions, particularly the take-off. Design charts that graphically correlate the various propeller parameters were prepared to facilitate the solution of problems and also to clarify the conception of the relationships of the various engine-propeller design factors.

It is shown that, among the many methods for improving the take-off thrust, the use of high-pitch, large-diameter controllable propellers turning at low rotational speeds is probably the most generally promising. With such a combination the take-off thrust may be further increased, at the expense of a small loss in cruising efficiency, by compromise designs wherein the pitch setting is slightly reduced and the diameter is further increased. The degree of compromise necessary to accomplish the maximum possible take-off improvement depends on such design factors as overspeeding and overboosting at take-off as well as depending on the design altitude. Both overspeeding and designing for altitude operation have the same effect on the take-off thrust as compromising in that the propulsive efficiency is increased thereby; boosting the engine, however, has the reverse effect on the propulsive efficiency, although the brake horsepower is increased.

For cases wherein the design basic pitch setting is necessarily low because of the inflexibility of the engine or airplane design relative to propeller speeds and diameter, there is little to be gained in performance from the use of special devices such as controllable propellers, but the advantage increases with the pitch setting or roughly, but not necessarily, with the speed.

There is no well-defined limit to the values of speed, power, and altitude to be attained in future propeller designs, but increasing each factor imposes increased design difficulties. Air speeds of the order of 400 miles per hour must be accompanied by pitch angles of at least 45° or

50°, and the diameters of 3-blade propellers may reach 15 feet for 3,000-horsepower engines for sea-level operation and 30 feet for 40,000 feet altitude.

## INTRODUCTION

The general conception of the engine and propeller as an integral propulsive unit is of fairly recent origin. Engine builders rarely invaded the propeller field to determine design factors affecting their product, and propeller manufacturers were usually content to build their products to fit the existing engines. Until the arrival of high-speed airplanes and controllable propellers, there was little need for cooperation as the best engine design features were fortunately often best for the propeller. High-performance airplanes, however, have brought special propulsive problems that require close study of the unit as a whole for the best solution.

This paper presents the results of a study of the relative efficiencies of various engine-propeller combinations covering the important flight conditions, particularly the take-off. Design charts are also given which graphically correlate the various propeller parameters.

## GENERAL DESIGN CHARTS

The familiar speed-power coefficient  $C_s$ , when plotted against  $V/nD$  and  $\eta$  for a family of propellers (see fig. 1), is very useful in designing or selecting propellers for given airplanes and engines. For conditions wherein the best engine gear ratio, blade width, or number of blades is undetermined, the designer must make additional computations before a propeller can be selected or designed. These extra computations often involve tip speeds, which definitely limit the product of the propeller rotational speed and diameter. In order to facilitate such computations and also to correlate the separate important propeller design dimensions on a single chart so that their relative values may more readily be visualized, a design chart (fig. 2) has been worked out based on the  $C_s$  chart given in figure 1. The  $C_s$  chart was drawn from data given in reference 1 and the curves were cross-faired in order to obtain a better working chart free from minor irregularities.

When  $C_s$  charts are used, the propeller is ordinarily designed to operate at high or cruising speed at a point on the envelope of the efficiency curves or at a point slightly below the envelope if the take-off and climb efficiency is to be increased. The latter design, involving reduced pitch and increased diameter, is generally known as a "compromise" design since the high-speed efficiency is slightly sacrificed in order to increase the efficiency at take-off. As the efficiency envelope curve or the corresponding compromise curve are the only  $C_s$  efficiency curves generally used, design charts may be made up with these two curves used as a basis. Figure 2 is such a chart based on the maximum-efficiency envelope curve (high-speed envelope curve) of a family of propellers for which the test results are given in figure 1. Figures 3 and 4 are additional sections A of the chart in figure 2 based on compromise curves, and may be superposed on section A of figure 2. The chart of figure 3 is for moderate compromise propellers of which the high-speed pitch setting is reduced  $3^\circ$ ; that of figure 4 is for a  $5^\circ$  blade-angle reduction at high speed. It has been found convenient to express the degree of compromise in terms of design blade-angle reduction for high speed.

Development of the chart (fig. 2):-- The chart of figure 2 is composed of a number of sections. Section A is the only part taken strictly from test results, the rest of the chart being only dimensional and therefore general for all propellers of somewhat similar construction. Section A may therefore be made up for different families of propellers as for different design conditions and may be used in conjunction with the rest of the chart for determining the various dimensional data.

$$\text{From the speed-power coefficient } C_s = \frac{0.638 \times m.p.h. \times \sigma^{1/5}}{h.p.^{1/5} \times r.p.m.^{2/5}},$$

where  $\sigma = \frac{\rho}{\rho_0}$ , the numerator (airplane function) is used

as the abscissa and the denominator (engine function) is used as the ordinate for plotting in section A values of propulsive efficiency, tip speed, pitch setting, and

$C_p^{1/5}$ , all of which are determined from the  $C_s$  envelope curve for maximum efficiency. The lines for constant efficiency are straight except at values corresponding to high tip speeds where they curve owing to tip-speed corrections (references 2 and 3). The lines for constant

$C_p^{1/5}$  are used for the determination of the propeller diameter as will later be more fully explained. While the straight portions of the lines for constant efficiency  $C_p^{1/5}$ , and pitch setting may readily be determined by solving for one point on each line, the lines for constant tip speed are more difficult to obtain, especially if the forward air-speed component is considered.

In the determination of the lines for constant tip speed, values of  $r.p.m. \times D$  were first obtained for different values of air speed at sea level, and  $V/nD$  was then determined. Since there is a definite value of  $C_s$  for each value of  $V/nD$ ,  $C_p^{1/5}$  was determined, using the relation  $C_p^{1/5} = \frac{V/nD}{C_s}$ . With the air speed at sea level given, the values for constant tip speed were plotted, either in relation to the  $C_p^{1/5}$  lines, or against  $hp.^{1/5} \times r.p.m.^{2/5}$  which equals  $\frac{r.p.m. \times D \times C_p^{1/5} \times \sigma^{1/5}}{138}$ .

138

The efficiency curves were corrected for tip speeds above approximately 1,000 feet per second. (See references 2 and 3).

Although section A of the chart alone (fig. 2) might be useful, it was believed that a complete expansion of both the abscissa and the ordinate so that each component might be read directly would apply more widely and would give a clearer picture of the relationship between the various parameters. The abscissa  $m.p.h. \times \sigma^{1/5}$  was consequently broken down into lines of constant air speed for different altitudes (section B). The ordinate  $hp.^{1/5} \times r.p.m.^{2/5}$  was also resolved into lines of constant propeller rotational speed for different values of engine power (section C).

Since the total blade width or the number of blades is also a function of the power, the power was further resolved into lines of constant power for different total blade widths (section D). It was assumed that the power absorbed by the propeller varied with the total blade-width ratio according to information given in figure 66 of reference 3. Inasmuch as the propulsive efficiency ordinarily decreases slightly as the total blade width is increased,

an arbitrary correction, section G, was included. This correction was taken from figure 66 of reference 3 but was slightly modified in view of more recent tests.

In order to develop the propeller-diameter chart, it was first necessary to resolve the ordinate

$C_P^{1/5} \times r.p.m. \times D \times \sigma^{1/5}$  into lines of constant  $C_P^{1/5}$   
 138  $\times r.p.m.$  for different values of  $D \times \sigma^{1/5}$  (section E), and then to plot lines of constant diameter for different altitudes in section F. It should be noted that the product of  $r.p.m.$  and  $C_P^{1/5}$  must be found before proceeding to sections E and F.

The efficiency correction for propeller-body interference is given in section H. This correction, recommended in reference 3, reduces the propulsive efficiency 1 percent below that for the test results for an 8-percent decrease in propeller-body-diameter ratio. Corrections for increases in propeller-body-diameter ratio are not recommended since tests indicate that little change in efficiency results from decreasing a nominal body size from 0.4 the propeller diameter to zero.

Tip speeds.- Unfortunately it was not possible to incorporate a complete set of tip-speed lines for all altitudes in the chart. Lines for 1,000 feet per second for the high-speed-propeller design at several altitudes were, however, included. Tip speeds for different altitudes may be readily obtained from the chart by determining the sea-level tip-speed value at the same  $C_P^{1/5}$  and air speed as those for the altitude under consideration. The tip speed thus obtained is the same as that for the altitude investigated since the tip speed is determined by  $V/nD$  and air speed, which remain constant under the foregoing assumption. In order to check any value more accurately, the tip speed may be computed from the relation:

$$\text{tip speed} = \frac{\pi \times r.p.m. \times D}{60 \cos \varphi_t}$$

where  $\cos \varphi_t$  is a factor that takes into account the forward velocity. Figure 5 gives values of  $\cos \varphi_t$  for different pitch angles.

It may be noted that the efficiency curves have been corrected only for the propellers operating at sea level. Since the propulsive efficiency starts to decrease at tip speeds of about 0.9 the speed of sound, the change in the speed of sound with changes in altitude should be taken into account for altitude designs. Figure 6 indicates that the velocity of sound decreases at the rate of 4.17 feet per second per 1,000 feet altitude up to the stratosphere. Stratosphere balloonists have reported increasing temperatures with increasing altitude above the tropopause, which indicates a reversal in the trend of the speed of sound at those altitudes.

It is not likely, however, that the speed of sound at altitude will determine the limiting condition, since the cruising propeller speed is ordinarily considerably lower than that for take-off. When checking the take-off tip speed, the factor  $\cos \varphi_t$  may be neglected because the forward velocity is small.

Use of the charts.— The following directions for the use of the charts apply to a composite chart composed of parts, A, B, C, D, E, and F combined as shown in figure 2(a). Extra copies of the three sheets that constitute the main chart may be obtained upon request from the N.A.C.A. in Washington. As the charts may be used for a variety of design problems, only general directions need be given. Read from section to section vertically or horizontally except across doubled division lines. The problem  $C_p^{1/5} \times r.p.m.$  must be solved before the diameter can be read from section F.

Example 1:

Given:      Design high speed,      200 m.p.h.  
                 Engine power,              700 hp. at 10,000 ft.  
                 Propeller speed,          1,400 r.p.m. for high  
     speed at critical altitude,  
                 Tip speed,                1,000 ft./sec. or less.  
                 Engine diameter,        54 in.  
                 Propeller designed to operate at peak effi-  
     ciency at high speed at critical altitude.  
                 To find; Diameter, pitch setting, and efficiency.

Solution: The vertical line through sections A and B is established in section B by the intersection of the line for 200 miles per hour and the line for 10,000 feet altitude. If a 2-blade propeller similar to 4412 is assumed, the vertical line through sections D and C is established in section D by the intersection of the 700-horsepower curve and the dotted line for the 4412 2-blade propeller. This vertical line intersects with the r.p.m. curve in section C at a value of  $hp^{1/5} \times r.p.m.^2/5 = 67.5$  and establishes the horizontal line through sections C, A, and E. Reading the values of pitch setting, efficiency, tip speed, and  $C_P^{1/5}$  in section A at the intersection of the vertical and horizontal lines: pitch setting =  $28.2^\circ$ ,  $\eta = 85$ , tip speed = 925 ft./sec. (approximately), and  $C_P^{1/5} = 0.588$ . In determining the diameter,  $C_P^{1/5} \times r.p.m. = 824$ . In section E the intersection of the horizontal line and the  $C_P^{1/5} \times r.p.m. = 824$  line determines a diameter of 12.0 feet. Since the propeller-body diameter ratio is  $12.0/4.5$  or 2.67 there is no efficiency correction. If a 4412 3-blade propeller is assumed, the horizontal line through sections C, A, and E is established at a value of  $hp^{1/5} \times r.p.m.^2/5 = 63$ , determining: pitch setting =  $29.8^\circ$ ,  $\eta = 84.2$ , tip speed = 865 ft./sec. (approximately),  $C_P^{1/5} = 0.595$ , and diameter = 11.1 feet. Since the propeller-body ratio is 2.47, there is no efficiency correction. The efficiency should be decreased about 2.5 percent, however, because of the total blade-width correction, resulting in a net efficiency of 81.7.

Example 2:

Given: Design high speed, 200 m.p.h.  
 Engine power, 700 at 10,000 ft. altitude.

Tip speed, 900 ft./sec. or less.

To find: Best propeller speed, best total blade width, diameter, and efficiency.

Solution: The intersection of the vertical line through sections A and B with the line for 900 ft./sec. tip speed determines the horizontal line through sections A, C, and E at a value of  $hp^{1/5} \times r.p.m^{2/5} = 65.0$ . For a 4412 2-blade propeller, the r.p.m. = 1,290, diameter = 12.6 feet, and the pitch angle = 28.9°.

If a lower tip speed is chosen, the engine-propeller combination will be characterized by a lower propeller rotational speed, a larger diameter propeller, a higher engine reaction torque, and a higher pitch propeller. If a controllable propeller is used, the higher pitch, larger diameter propeller will result in improved propulsive efficiency at take-off, as will later be more fully explained. In view of the higher efficiency at take-off, a 37° pitch angle propeller might be selected, which would define a horizontal line through sections A, C, and E at a value of  $hp^{1/5} \times r.p.m^{2/5} = 47.8$ . For a 4412 2-blade propeller, the r.p.m. = 600 and the diameter = 18.6 feet.

This example illustrates the use of the chart in rapidly setting up the boundaries of design problems.

#### COMPARISON OF VARIOUS ENGINE-PROPELLER COMBINATIONS

Various engine-propeller combinations have been studied with the purpose of evaluating their relative thrust-producing effectiveness for all flight conditions. The propeller test data used as a basis for all computations are given in figure 1. The reference tests were made at various pitch settings from 17° to 42° of a family of 4-foot model propellers tested in conjunction with a radial-engine nacelle mounted on a section of a thick wing.

An examination of the envelope propulsive efficiency curves reveals that the peak efficiency occurs at a pitch setting of  $27^\circ$ , which is believed to be a characteristic of this particular test set-up and not necessarily to hold for propellers in general. Tests reported in references 4 and 5 do not indicate a decreasing efficiency up to pitch settings of even  $36^\circ$  or  $40^\circ$ . Furthermore, full-scale tests (see appendix) made of several late propeller designs indicate that the envelope efficiency curve remains fairly flat up to about  $40^\circ$  pitch setting. The relative values at the take-off and climb are not altered, however, by any such unique characteristics of the propeller data used.

Most of the computations to determine the relative effectiveness of engine-propeller combinations were based on tests of unsupercharged engines; some of the computations, however, were repeated for supercharged engines.

Although the relative efficiencies of the various engine-propeller combinations have been worked out for the entire speed range, emphasis has been placed on the four most important flight conditions: take-off, climb, cruising, and high speed. A representative take-off speed has been assumed, equal to 0.3 the high speed, which corresponds to a speed at the latter part of the take-off run for average airplanes. It is shown in reference 6 that it is possible to compute the take-off run fairly accurately by using the excess propeller thrust at 0.7 the take-off speed, which corresponds to 0.3 the high speed for airplanes having a speed range of about 2.5 or 3.0. The climbing speed has been assumed to be equal to 0.65 the high speed. The cruising criterion is the same as that for high speed since propellers generally operate at about the same value of  $V/nD$  for both conditions.

#### Unsupercharged Engines

The engine torque was assumed to remain constant for small changes in engine speed at full throttle for unsupercharged engines.

Fixed-pitch propellers.— In figure 7 the ratio of the thrust horsepower to brake horsepower available at high speed,  $t.\text{hp.}/b.\text{hp.}_m$ , is given for various pitch settings and all air speeds up to the high speed. Inasmuch as the full-throttle engine speed decreases with the air speed,

the ratio  $t.\text{hp.}/b.\text{hp.}_m$  is equal to  $\eta \times \frac{r.p.m.}{r.p.m._m}$  where

the subscript  $m$  denotes the condition at high speed. It is noteworthy that even though the ratio  $t.\text{hp.}/b.\text{hp.}_m$  varies somewhat for the different pitch settings at high speed, all the values for the latter part of the take-off run fall within a few percent of each other. This agreement indicates that, as far as pitch-angle selection is concerned, the best propeller for high speed will probably also be the best for take-off; compromise designs are, of course, neglected.

A comparison has been made in figure 8 of a high-speed and a compromise design. The results are given for a wide range of design  $C_s$  values and show the difference in thrust horsepower available for the four important flight conditions. It is interesting to note that reducing the design blade setting  $3^\circ$  and increasing the diameter slightly to compensate increases the take-off and climbing thrust a small amount and slightly reduces the high-speed thrust.

Compromising with fixed-pitch propellers is not very effective because the loss in engine power resulting from lower engine speeds nearly offsets any gain in propulsive efficiency. Figure 9 shows the change in full-throttle engine speed with change in air speed for high-speed and compromise design propellers of various basic pitch settings; it further shows that the propeller speed at low speeds decreases with increasing pitch setting up to about  $27^\circ$  but, for higher pitch settings, further decreases in propeller speed are not so pronounced.

Fixed-pitch and variable gear ratio. - In this analysis it was assumed that the engine gear ratio could be changed through an infinite number of stages so that the engine speed would remain constant for all air speeds. By such a hypothetical arrangement the full-throttle engine power would remain constant and the propulsive efficiency would differ but little from the ungeared condition. In figure 10 the results of computations are given for propellers designed for maximum efficiency and also for  $3^\circ$  blade-angle reduction, compromise propellers with variable gear ratio. The chief merit of a variable gear ratio is the ability to maintain the engine power for the take-off since little change in propulsive efficiency can be expected. The compromise propeller therefore nets a

greater gain in take-off thrust with than without a variable gear ratio because the propulsive efficiency is increased and there is no loss in engine power.

Since infinite variable gear ratios are not practicable at present, any contemplated designs must be confined to a finite number of gear changes. Actually, one gear change would probably suffice since the propeller speed need be reduced only for the take-off and climb.

Controllable pitch.- There are at least three important advantages of controllable propellers, especially of the constant-speed type: The pitch angle may be set to allow the engine to turn at the desired speed, thereby creating the most advantageous engine-operating conditions; the propulsive efficiency for take-off and climb increases with the design basic pitch settings over that for fixed-pitch propellers; and compromise designs result in greater gains than fixed-pitch propellers, especially for the high basic pitch settings.

For basic pitch angles below about  $20^{\circ}$ , the first advantage is the only one that is important. Above basic pitch angles of  $20^{\circ}$ , the last two advantages become more and more pronounced.

Figure 11 illustrates the improvement in thrust power with increased basic pitch setting for propellers designed for maximum efficiency at high speed. When the differences in high-speed efficiency are neglected, it is quite evident that the higher the pitch setting the higher the thrust up to some limiting pitch-setting value of about  $40^{\circ}$ . If a family of propellers having a fairly flat envelope of the efficiency curves, such as those reported in the appendix of this paper had been used in this analysis, the high-pitch propellers would have given the best all-round performance. High pitches will, of course, require a low speed, which in turn necessitates a large diameter for any given airplane.

In order to illustrate the operating characteristics of various compromise designs, a plot of propulsive efficiency against air speed for an airplane used as an example is given in figure 12. In this figure the solid curves are lines of constant pitch and are included only to show the actual propeller operating pitch setting represented by the dotted lines. If a controllable propeller that

would have its maximum efficiency at the high speed (250 miles per hour) were selected, it would represent the no-compromise condition, and the efficiency would follow the dotted curve for 0° blade-angle reduction at different air speeds. If a propeller of 3° lower blade angle, and consequently of greater diameter, were selected, the high-speed efficiency would be slightly less, but the efficiency at the latter part of the take-off run and at the climbing speed would be considerably higher. Still greater improvement in the take-off efficiency may be made by lowering the pitch setting and enlarging the propeller diameter to a limiting condition wherein the operating curve touches the envelope of the efficiency curves at the take-off speed. The high-speed efficiency, of course, progressively decreases with each improvement in take-off efficiency but in much smaller proportion.

The effects of different amounts of blade-angle reduction are given for various pitch settings and design values of  $C_s$  in figure 13. It is apparent that for basic pitch settings below about 17° there is no object in attempting to compromise but, as the pitch setting increases, so does the possibility of improvement in take-off thrust. For high basic-pitch propellers the take-off thrust may be increased by as much as 40 percent by this blade-angle reduction compromise feature alone. The gain in climbing thrust power increases noticeably for the 3° blade-angle reduction, but little further gain is possible. The high-speed or cruising efficiency drops slightly for each degree of blade-angle reduction and at an increasing rate.

Under the section dealing with supercharged engines, it will be shown that certain engine characteristics influence the degree of compromise necessary or advisable.

Figure 14 shows the increase in diameter necessary to compensate for various degrees of blade-angle reduction for compromise designs.

Among the various objections to the large-diameter propellers that may result from increasing the design pitch setting and compromising, weight is perhaps the only one affecting the take-off and climb characteristics aside, of course, from the thrust. If other factors are neglected, it is advantageous to increase the pitch setting and diameter only to the point of best performance.

From the relationship derived for a given airplane:

$$\frac{S_a}{S_b} = \frac{W_a}{W_b} \frac{(T_e)_b}{(T_e)_a}$$

where  $S$  is the take-off distance,

$W$ , the gross weight,

$T_e$ , the excess thrust at 0.7 the take-off speed,

and the subscripts  $a$  and  $b$  denote the two conditions.

It is evident that the best propeller for take-off is the one giving the shortest distance  $S$  or the greatest excess thrust per unit gross weight. Since the excess thrust changes rapidly with total thrust and the gross weight changes slowly with propeller weight, the thrust is far the more important factor.

In order to illustrate the effect of increasing the design pitch on the take-off distance, an example condition (fig. 15) is included. In this illustration the take-off distance is given for a typical twin-engine transport having controllable propellers of various design pitch settings and diameters. The propeller weights were assumed to be proportional to the cube of the diameter.

It may be noted that increasing the design pitch setting from  $25^\circ$  to  $40^\circ$  decreased the take-off distance about 29 percent. This percentage is not materially altered even though the propeller weight, which was greatly increased, is neglected.

Similarly, relative rates of climb may be found using the relationship:

$$\frac{C_a}{C_b} = \frac{(T_o)_a}{(T_o)_b} \frac{W_b}{W_a}$$

where  $C$  is the maximum rate of climb,

and  $T_o$ , the excess thrust at the climbing speed.

Controllable pitch and diameter.— Up to the present time no satisfactory method has been devised to change the

propeller pitch setting and diameter while in flight, nor does it seem likely that there soon will be. As such a device would represent the upper limit in propeller efficiency, however, it will be briefly discussed.

In the previous section on controllable propellers it was shown that, when the design pitch setting was reduced and the diameter increased, the efficiency at low speeds was improved but the high-speed efficiency suffered. Obviously, if the pitch setting and diameter could be simultaneously changed the operating curve would coincide with the envelope of the efficiency curves.

In figure 16 the operating curves are given for several design basic pitch settings. Similar to but more pronounced than controllable propellers, the thrust at take-off and climb increases with pitch setting.

As a matter of academic interest the change in diameter with change in air speed is given in figure 17 for controllable pitch and diameter propellers. A propeller capable of having its diameter changed the required amount is entirely possible but, since the performance of compromise controllable propellers nearly equals that of controllable pitch and diameter propellers, their advantages do not merit the added complications for the ordinary case.

Controllable pitch with one or two gear changes.— Since compromise controllable propellers are slightly handicapped at high speed, a combination controllable pitch and gear change might be preferable for some installations. If the high-speed or cruising propeller were designed to operate on the low-speed gear, the maximum efficiency could be obtained for those flight conditions. If the gear ratio were changed to overspeed the propeller at take-off and the pitch reduced to compensate, the efficiency for take-off could be raised to that obtained by the compromise design or even by the controllable pitch and diameter if sufficient gear speed increase were provided. This method makes it possible to obtain high efficiency for both take-off and high speed but imposes the complication of the gear change. Thrust-power curves are given in figure 18 for two different gear changes. It should be noted that overspeeding the propeller increases the tip speed as does increasing the diameter for compromise propellers. In either system the tip speed should not exceed 1,000 feet per second, else the efficiency will drop and noise and vibration will increase. (See reference 2.)

Comparison between 2- and 3-blade propellers. - It might be reasoned that, since an increase in total blade-width ratio would induce a greater inflow velocity at the take-off, the blades would be operating at a smaller angle of attack and the stall would occur at a lower air speed. This reasoning leads to the conclusion that wide or 3-blade propellers might be better for take-off than narrow or 2-blade ones, but neglects the conception of the ideal efficiency which indicates that the greater the slip-stream velocity the lower the efficiency. Also, if the diameter is reduced for the propeller of greater total blade width, the ideal efficiency is reduced.

Test results conclusively proving or disproving the hypotheses are not at present available, but some high-speed small-scale tests (reference 4) indicate that 2-blade propellers are superior to 3-blade ones for nearly all conditions (fig. 19). A check comparison has been made of some recently tested full-scale 2- and 3-blade propellers, which confirms the conclusions drawn from figure 19. (See fig. 37 of the Appendix.) If increasing the blade width is construed as having the same effect as increasing the number of blades, as is indicated in reference 3, the same conclusions apply to changes in blade width.

Comparisons between various unsupercharged engine-propeller combinations. - In the preceding sections various engine-propeller combinations were discussed individually but few comparisons were made. In order to obtain a better understanding of the relative values of the various methods described, comparative curves have been prepared. In figure 20(a) thrust horsepower curves are given for the moderately low basic pitch setting,  $22^\circ$ , while figures 20(b) and 20(c) are for  $32^\circ$  and  $42^\circ$  pitch settings respectively; pertinent data are also given in table I.

Several points of interest may be noted from figure 20(a). At a speed of  $0.3 V_m$  (the take-off criterion), the propeller with controllable pitch and diameter provides the greatest possible gain in thrust, about 33 percent, over the fixed-pitch propeller. Of this possible 33 percent increase the infinite variable gear ratio produces only about one-third, the controllable pitch designed for maximum speed about half, and the  $3^\circ$  compromise controllable propeller about 29 percent. Since the compromise controllable propeller so closely approaches the upper limit, there is little need to seek new methods to improve the thrust for this relatively low-pitch propeller.

If it were necessary to increase the take-off thrust above the possible limit for the  $22^\circ$  pitch-setting propeller, it would be necessary to increase the basic pitch because the upper limit at take-off increases as the basic pitch increases. This fact is brought out by comparing the curves of figures 20, (a), (b), and (c). Whereas only 45 percent of the possible brake horsepower is available at take-off for the  $22^\circ$  basic pitch setting, 61 percent is available for  $32^\circ$ , and 71 percent for  $42^\circ$ . The reason for this large possible increase is that with higher pitches the propeller speed must be decreased; with the lower propeller speed the diameter must be increased; and with the larger diameter the ideal efficiency is increased. Even though the propeller may be stalled at the take-off, the higher efficiency resulting from the larger propeller more slowly moving the greater mass of air more than compensates for the higher profile drag of the blade sections.

Increased-diameter propellers resulting from increased pitch settings and decreased propeller speed should not be confused with increased-diameter propellers resulting from reduced pitch settings and constant propeller speed, as is the case for compromise designs. In the former case the high-speed efficiency is taken at a higher value of  $C_s$  from the envelope of the curves and the tip speed is decreased; whereas in the latter case the  $C_s$  remains about constant, the high-speed efficiency is reduced below the envelope, and the tip speed is increased. The effect on the take-off efficiency is qualitatively the same in either case and the effects may be quantitatively added. The net result of both of these effects accounts for the large differences in take-off thrust available with compromise propellers of different pitches. (See fig. 20.)

It may be noted from figure 20(b) or (c) that a controllable propeller with an overspeed drive with a ratio of 1.25 is nearly equivalent to the  $5^\circ$  compromise controllable propeller. The essential difference between these two systems is that the propeller speed is increased in one case and the diameter in the other. Both systems are characterized by a slight efficiency drop at high speed and by increased tip speeds. Since the overspeed drive would also incorporate a single gear change, the gears would be shifted to a lower ratio for high speed and cruising. Although a gear change would avoid the loss in efficiency at high speed, the mechanical difficulties and the weight would be increased.

A single gear change with a fixed-pitch propeller might be preferable to some of the other combinations for light airplanes because of economic rather than aerodynamic reasons if it were necessary to improve the take-off thrust. A single gear change and a wooden propeller would probably be lighter and definitely cheaper than a metal controllable propeller and the performance of low-speed airplanes would be only slightly inferior if the fixed-pitch propeller were of the compromise type. Not only could the engine speed be increased to normal for take-off but it could be overspeeded, thereby gaining additional efficiency and engine power. Also, if reduction gearing were used, the propeller-engine combination could be designed for a large, high-pitch propeller giving peak efficiency at high speed.

#### Supercharged Engines

In the section dealing with unsupercharged engines it was assumed that the engine torque remained constant for all full-throttle flight conditions and also that the engine speed never exceeded a certain value even at the take-off. Supercharged engines, however, are characterized by having a range of torque values corresponding to different manifold pressures, different engine-speed ratings for different flight conditions, and the ability to maintain the normal operating manifold pressure up to a given altitude. Each of these factors affects the propeller design and also the relative merits of the various engine-propeller combinations. Boosting the engine for take-off improves the take-off thrust of fixed-pitch propellers by virtue of both the increased engine speed and the torque, while boosting is less effective with controllable propellers because some propulsive efficiency is lost owing to the higher pitch required to absorb the additional torque. The practice of overspeeding supercharged engines fitted with controllable propellers is effective in increasing the take-off thrust because both the propulsive efficiency and the engine power are increased.

Since fixed-pitch propellers for supercharged engines are usually designed to absorb the normal full-throttle power at the critical altitude, there will be a pronounced drop in engine speed at sea level for the same engine torque. (See fig. 21.) This decrease is due to the larger diameter necessary for the altitude operation. Overboosting at take-off decreases the engine-speed drop, but a great deal more boosting than is allowed by engine manufac-

turers at the present time is required to raise the engine speed up to its normal rated value. Figure 21 indicates that full-throttle operation at sea level of the engine assumed (46 inches of mercury manifold pressure) will only raise the engine speed up to values equaling that for an unsupercharged engine. If it were possible to boost the engine to the point where the engine speed at take-off would equal the desired value, there would be no advantage in using a controllable propeller, at least, for take-off because the pitch setting would be the same as that for high speed. As only a small overboost is allowed at present for take-off, it is quite evident that controllable propellers are more useful with supercharged engines than with unsupercharged engines because of the greater increase in take-off engine power of the former.

Comparisons between various supercharged engine-propeller combinations.— The relative merits of various engine-propeller combinations are brought out in figure 22, and in table I for a supercharged engine having a critical altitude of 7,000 feet. The results are given in the form of a ratio of thrust horsepower available to the maximum brake horsepower at the critical altitude  $t.\text{hp.}/b.\text{hp.}_{mc}$  for different ratios of the air speed at sea level to the high speed at the critical altitude  $V/V_{mc}$ . Plots have been made for three engine ratings: high-speed and climb, take-off, and full-throttle. All these conditions are for sea-level operation since the take-off and climbing conditions bring out the important differences of the various combinations. The full-throttle condition is somewhat beyond present-day practice but is included to show the effect of boosting to extremely high pressures. Advances are continually being made in engine design and fuels, however, and it is not unlikely that before long full-throttle operation at sea level may be possible for short periods. The engine used for these computations is a typical radial air-cooled engine moderately supercharged (7,000 feet). The results, therefore, will differ quantitatively for engines of different degrees of supercharging but will show the same general effect.

Figure 22(a) shows the results for relatively low-pitch propellers ( $22^\circ$  basic pitch setting at high speed at the critical altitude). The poor performance shown by the fixed-pitch propeller is quite noticeable for the take-off and climb ratings but less so for the full-throttle condition.

The variable gear ratio combination appears to be better than it did for the unsupercharged-engine combination because the decided engine-speed drop at low speeds and altitude, which is responsible for the lower fixed-pitch performance, is eliminated by the variable gear ratio feature.

It is important to note that the controllable-propeller curve nearly coincides with the controllable pitch and diameter curve for the take-off and climb ratings. Figure 20(a) shows that the controllable propeller designed for maximum efficiency at high speed does not show such relatively high values for the unsupercharged engine but that the 3° compromise design does. This fact indicates that propellers designed for supercharged engines require less compromise to accomplish the same relative take-off efficiency than do propellers designed for unsupercharged engines.

The effect of overspeeding and overboosting on the take-off thrust power is quite marked. Increasing the manifold pressure from 35 to 37.5 inches Hg and the engine speed from 1,950 to 2,050 r.p.m. increases the take-off thrust about 10 percent, while opening up the throttle increases the thrust 37 percent (values taken from the controllable-pitch and diameter curves).

The effects enumerated for the 22° blade-angle propellers are, in general, the same but more pronounced for the 32° and 42° propellers (figs. 22(b) and (c)). It should be noted that the higher the basic pitch setting, the greater becomes the possibility of improving the take-off by means of the use of controllable propellers. Whereas the take-off thrust increases 47 percent with a controllable propeller of 22° pitch setting for the take-off engine rating, it will increase about 109 percent with a 3° compromise propeller of 42° basic pitch setting. The increase is about in the same proportion as it was for unsupercharged engines.

It may be seen that, while the 3° compromise design curve approaches the controllable pitch and diameter curve fairly closely for the take-off rating, it approaches the limiting curve less closely for the full-throttle condition. This fact indicates that overboosting requires a greater degree of compromise to give the same relative efficiency than when no boosting is used.

Compromise designs for supercharged engines.— It has

previously been brought out that the degree of compromise necessary with controllable propellers to obtain the highest possible take-off thrust depends upon: the propeller basic pitch, the design altitude, the amount of engine overspeeding at the take-off, and the amount of overboosting. Figure 13 shows that the higher the basic pitch, the greater is the possible take-off improvement and the higher the degree of compromise necessary to obtain the increased thrust. This generalism applies to all types of engines; but the special supercharged-engine characteristics change the magnitude of the desired compromise according to the design altitude, the amount of overspeeding, and the amount of overboosting.

It is readily seen that the pitch must be reduced when going from a region of rarefied atmosphere at altitude into the denser atmosphere at sea level in order that the propeller absorb the same power, other factors remaining constant. This pitch reduction has the same effect on the take-off efficiency as the same reduction made as a compromise but, of course, does not reduce the high-speed efficiency. The pitch must be further reduced for any overspeeding of the engine at take-off. Boosting the engine for take-off, however, has the opposite effect because the pitch must be increased to absorb the extra power. The effect of each of these three factors on the equivalent compromise pitch reduction is shown in figure 23. The chart is read up from each side and across from side to side.

The chart shows that, if there is no change in power or engine speed, the altitude designs have the same effect on take-off as do compromise propellers. For example, it may be noted that, for an airplane having a critical altitude of 10,000 feet (condition A), the blade angle must be reduced  $3\frac{1}{2}^{\circ}$  for the same air speed at sea level, resulting in an increase of approximately 20 percent in take-off thrust for the high-speed airplanes. (See fig. 13.) Also, if the power at constant engine speed is increased by 10 percent (condition B), the blade angle must be increased  $1^{\circ}$  (from A to B) with the result that the improvement in take-off efficiency will be reduced to approximately 14 percent. Since the engine power is increased 10 percent, the net increase will be about 25 percent as compared with 32 percent had there been no loss in propulsive efficiency due to the added engine torque. On the other hand, if the engine is oversped by 5 percent (condition C), the blade angle must be reduced about  $1\frac{1}{2}^{\circ}$  (from A to C) or

a total reduction of about  $5^{\circ}$  for the same speed, resulting in a more than 30-percent increase in take-off thrust power for high-speed airplanes as compared with 20-percent increase for condition A.

It is quite evident, therefore, that if engine temperature, or power for a short time, is the limiting factor in rating engine for take-off, it is better to obtain that power at the highest possible engine speed rather than at a high manifold pressure and a low engine speed.

It may be noted from the chart that propellers designed for high-altitude flight will have very good take-off characteristics since the pitch must be reduced considerably for sea-level operation.

The suggested procedure for selecting propellers for supercharged engines is: (a) From figure 23 determine the blade-angle reduction necessary for sea-level operation at the take-off engine rating; (b) from figure 13 determine any additional compromise blade-angle reduction; (c) from figures 2, 3, 4, and 5 determine the propeller characteristics.

Example 3:

Given: Engine 800 horsepower at 10,000-foot altitude and 900 horsepower for take-off.

Propeller speed, 1,200 r.p.m. (normal).

Propeller speed, 1,260 r.p.m. (take-off).

Airplane speed, 250 miles per hour at 10,000 feet.

Solution:

(1) From figure 23 an increased power ratio of 1.125 and an increased propeller-speed ratio of 1.05 results in a blade-angle reduction of  $3.3^{\circ}$  in sea-level flight at the same speed.

$$(2) C_s = \frac{0.638 \times m.p.h \times \sigma^{1/5}}{(0.7 \text{ hp.})^{1/5} \times r.p.m.^{2/5}} \text{ (for 3-blade propellers)}$$

$$= \frac{0.638 \times 250 \times 0.942}{3.555 \times 17.04} = 2.48$$

(3) From figure 13 it may be noted that little is to be gained by reducing the blade angle beyond  $70^\circ$  for a  $C_s$  value of 2.48. If a total blade-angle reduction of  $6.3^\circ$  is assumed, the blade angle need be reduced only  $3^\circ$  below the basic design since  $3.3^\circ$  is cared for by the engine characteristics.

(4) From figures 2 and 3 the pitch setting is  $33.9^\circ$ , and the diameter is 12.5 feet for a 3-blade propeller. From figure 5 the tip speed is 870 feet per second for high speed at altitude or 830 feet per second for take-off. The high-speed efficiency is 0.775 when corrected for solidity. The  $t.\text{hp.}/b.\text{hp.}m_c$  at 0.3 high speed or at the latter part of the take-off run may be computed from figure 13 reasonably well, or more accurately from figure 1. At a value of  $C_s$  of 2.48 the  $t.\text{hp.}/b.\text{hp.}m_c$  is 0.625 for the  $6.3^\circ$  compromise propeller. Since the take-off power is increased from 800 to 900 horsepower, the  $t.\text{hp.}/b.\text{hp.}m_c$  is  $0.625 \times \frac{900}{800}$  or 0.704. This value should also be corrected for solidity leaving 0.68 for the final  $t.\text{hp.}/b.\text{hp.}m_c$ .

It should be remembered that all the charts in this paper are based on propeller data taken from reference 1 and should be used with discretion for problems involving other propeller shapes.

#### RELATIONSHIP BETWEEN PROPELLER SPEED, DIAMETER, AND PITCH FOR DIFFERENT SERVICE CONDITIONS

In order to obtain the best all-round efficiency from the engine-propeller unit, it is obviously necessary to determine the best propeller speed, diameter, pitch setting, and number of blades. Since there are so many theoretical and practical considerations, it is impossible to determine

these dimensions for the field of airplanes as a whole except in a very general way. Certain elements of performance of one type of airplane may not be considered important for another. In some designs the geometry or dimensions of an airplane might determine the propeller size. Engine speeds and propeller diameters also are more or less fixed by the makers because it is impossible to furnish infinite variations of these quantities.

Entirely satisfactory methods for evaluating the proper engine-propeller parameters for the field as a whole appear impossible to determine; however, there is included herein an attempt to show the relationships between certain design dimensions based on various assumptions.

Propeller speed - diameter charts. - Figure 24 is a chart for obtaining the best propeller speed and diameter for the maximum high-speed efficiency for sea-level designs. The assumptions used as the basis of the chart are: (a) The propeller is to operate at maximum efficiency at high speed, and (b) the take-off efficiency is of secondary importance. It is not possible to include the altitude function in the chart because of the differences in diameter and tip speed with changing height. The chart is of limited application because the take-off is ordinarily of major importance. It is interesting to note that the propeller speeds are, in general, fairly close to those used in present-day practice, especially if it is considered that the values may be considerably increased or decreased without appreciably affecting the efficiency. The propeller speed increases with air speed and decreases with power, while the reverse is true for the diameter.

For conditions in which the propeller diameter is limited by such factors as weight, ground clearance, fuselage clearance, engine-reaction torque, or propeller cost a chart has been prepared based on arbitrary diameters equal to 2.5 times the average engine diameter (fig. 25). Such a scheme is practicable since the propeller stock sizes and the engine gear ratios are few. Of course neither the high-speed nor the take-off efficiency is necessarily the highest possible. With this design scheme the propeller speed decreases as the air speed increases, which is contrary to the results of figure 24.

The engine-diameter-horsepower relationship was determined from figure 26, which is a plot of a number of pres-

ent-day radial engines. The curve represented by the expression,  $\frac{\text{diameter}_1}{\text{diameter}_2} = \left(\frac{\text{hp.}_1}{\text{hp.}_2}\right)^{1/5}$ , was used since the chart could be simplified thereby.

It was previously pointed out that for all-round efficiency, it is desirable to use high pitches and large diameters for controllable propellers. It is fairly safe to assume that an arbitrary selection of a pitch setting of  $37^\circ$  will ordinarily give about the maximum efficiency at high speed. (The assumption is based on data given in the Appendix.)

This analysis shows that controllable propellers (especially compromise designs) of pitch settings of the order of  $37^\circ$  will also give about the best take-off thrust. It seems quite clear, then, that if the propeller speed and diameter were determined for a propeller of, for instance,  $37^\circ$  pitch setting, the over-all efficiency would be about the maximum. Figure 27 shows a chart for the determination of the speed and diameter of a propeller having a basic pitch setting of  $37^\circ$ . In this chart the altitude function is included, but the diameter for altitude designs must be computed from the relation, diameter =  $56 \times \frac{\text{m.p.h.}}{\text{r.p.m.}}$ . The chart is applicable for altitudes up to 40,000 feet and speeds up to 300 miles per hour. For higher speeds higher pitches must be used in order to keep the tip speeds within the limiting value of 0.9, the speed of sound.

In general, the chart resembles the one given in figure 24 except that the propeller speed is lower and the diameter higher for any given condition. With such low propeller speeds the engine-reaction torque will be high, and single-engine airplanes may be rendered uncontrollable for certain conditions. This objection is not so serious for large multiengine airplanes, however, especially if the net torque is reduced to zero by turning half the propellers in one direction and half in the other. Large diameters are objectionable for small craft from considerations of clearance and weight, but for large craft this objection is minimized. Large propellers may even be a definite asset for installations in which a large wing or fuselage is placed in the slipstream.

Large diameter propellers used on multiengine airplanes, of course, necessitate increasing the distance between the engines or between the engine and the fuselage, thereby increasing the difficulty of overcoming the unbalanced thrust couple when one engine fails in flight.

It may be mentioned that propellers selected by means of figure 27 will operate at the same tip speed for any given engine power regardless of the number of blades or diameter. In this chart the product of the propeller speed and diameter is constant for constant power. It is quite evident, therefore, that there would be no object in using three blades instead of two if it were necessary to reduce the tip speed. There might be some other advantage in using three blades, however, such as less vibration, less noise, or smaller diameter. Since the diameter for the 3-blade propellers is 0.845 times the diameter for 2-blade ones, the weight of the 3-blade propellers will be about  $1.5 \times (0.845)^3$  or about 0.91 times the weight of 2-blade propellers, assuming that the blades, shanks, and hubs are geometrically similar.

It should be noted that the charts given in figures 24, 25, and 27 give only the basic propeller dimensions, not necessarily the final dimensions. For conditions in which compromise designs are desirable, the pitch must be reduced and the diameter increased. The procedure recommended for using charts such as these is:

- (1) Determine the best engine speed from one of the charts (figs. 24, 25, and 27). The diameter may be tentatively determined to form a guide, but the final diameter may be slightly different.
- (2) Determine the blade-angle reduction necessary for sea-level operation from figure 23 if a supercharged engine is used.
- (3) Determine the additional compromise blade-angle reduction, if any, from figure 13.
- (4) Determine the propeller dimensions and tip speed from figures 2, 3, 4, and 5.

Example 4:

Given: 1,000 horsepower at 10,000 feet.

1,200 horsepower for take-off with 10 percent overspeed.

250 miles per hour high speed.

To find: Best propeller speed.

Best diameter.

Final pitch setting, etc.

Solution:

(1) From figure 27 the propeller speed is approximately 900, and the diameter is 15.5 feet for 2 blades; the propeller speed is 1,050 and the diameter is 13.3 feet for 3 blades.

(2) From figure 23 the blade-angle reduction for sea-level operation under take-off engine conditions is found to be  $3.6^\circ$ . (A speed of 250 miles per hour is, of course, assumed.)

(3)  $C_s = 2.50$  (2 blades).

The  $C_s$  should be the same for a 3-blade propeller since the pitch setting is  $37^\circ$  for both. From figure 13 it may be noted that  $3^\circ$  additional blade-angle reduction, as a compromise measure, will increase the take-off thrust to about the maximum amount obtainable with only a slight decrease in high-speed efficiency.

(4) From figures 2, 3, and 5:

For a 2-blade propeller, the efficiency is 80, the diameter is 16.7 feet, the pitch setting is  $34^\circ$ , and the tip speed is 875 feet per second for high speed or 867 feet per second for take-off. (Note the tip-speed decrease even though the engine is overspeeded for take-off.)

For a 3-blade propeller, the efficiency is 77.5, the diameter is 14.2 feet, and the pitch setting and tip speed must be the same as for the 2-blade propeller, neglecting errors.

Future design limitations.— There has been some concern shown by designers about the possibility that the advance of airplane design may be limited by the propeller. Higher speeds necessitate higher pitch settings; higher powers necessitate greater blade areas, either larger diameter or greater total blade width; and higher altitudes necessitate greater diameters.

The pitch is probably limited by virtue of the diminishing efficiency for the higher pitches. Since the theoretical maximum efficiency occurs at a pitch angle a few degrees below  $45^\circ$ , it is not unlikely that pitch angles up to  $50^\circ$  could be used without sacrificing a great deal of maximum efficiency. It has been shown in this paper that with controllable propellers the efficiency at a given percentage of the high speed, such as  $0.3 V_{max}$ , increases as the pitch angle increases at least up to  $42^\circ$ , the limit of the pitch angles investigated. Thus, the propeller operating conditions for take-off and climb are more favorable for high-speed than for low-speed airplanes. At the high and cruising speeds, 300-mile-per-hour airplanes are still in the high-efficiency range, as may be seen from figure 2, while 400-mile-per-hour airplanes operate beyond the peak of the theoretical curve; how much below the peak of the actual curve their operation is cannot at present be determined.

The power may be increased without affecting the tip speed  $KnD$  either by increasing the total blade width or by increasing the diameter and decreasing the rotational speed. If the former method is used, the propulsive efficiency suffers slightly; and if the latter method is used, the larger diameter propeller may be objectionable from considerations of weight or actual size. The lower propeller speed also results in a higher engine-reaction torque unless two propellers are operated in opposite directions. The limit to which the solidity may be increased is obviously imposed by the diminishing efficiency. The limiting diameter is determined by the weight or size that can be tolerated on any given airplane, the larger the airplane the larger the propeller that may be permitted.

Since the trend in airplane speeds parallels, in general, the trend in engine power, the increased power is absorbed to some extent by virtue of the higher pitches. In such cases the diameter may not change a great deal even though the power is increased. It appears, therefore, that for cases in which the increased power corresponds to the increased airplane size and speed the upper limit of the engine power that can be efficiently absorbed is very vague, but is certainly not under 2,000 or 3,000 horsepower.

The most noticeable result of the high-altitude factor on propeller design is the large diameter necessary. For example, if it is assumed that the high speed, 200 miles per hour, of a certain airplane will increase at the rate of 1 percent per 1,000-foot altitude up to 30,000 feet with constant engine power, 700 horsepower at 1,400 r.p.m., the diameter will increase from 10.4 feet to 12.3 feet for the 3-blade design. (See fig. 2.) This increased diameter and air speed will increase the tip speed 200 feet per second up to a maximum speed of 1,000 feet per second for the high altitude design. If the comparison is based on a constant tip speed, the engine speed must be reduced to 800 r.p.m. and the diameter increased to about 16.7 feet equivalent to a 60 percent increase in diameter. The important tip-speed value at sea level is that for take-off and at altitude is that for cruising, which is at lower propeller speeds. These differences in engine speed will affect the actual necessary diameter somewhat.

Another important effect of the high-altitude factor is the increased pitch accompanying the increased air speed. For the example cited, wherein the tip speed is held constant, the pitch angles increased from  $30.9^\circ$  to  $39.3^\circ$ .

It is quite evident from this example that the diameter and pitch setting increase at an alarming rate with increased altitudes if the tip speed is held constant. Since higher powers also affect the diameter in a similar way, the combination of both elements hastens the practicable limit in design.

In order to bring out in a more concrete manner the diameters and pitch settings that are likely to be encountered, table II has been prepared showing different air speeds, altitudes, and horsepower. It should be noted that the table is based on 3-blade propellers operating at a tip speed of 1,000 feet per second and therefore

represents a fairly high solidity value and about the upper limit in tip speed.

It is interesting to note that the diameter increases greatly with increasing altitudes and also with increasing values of power, but decreases slightly with increased air speed. The propeller speed is affected inversely as the diameter, and the pitch setting is entirely a function of air speed under the present assumptions.

For any given air speed and tip speed the  $C_s$  is constant; hence  $hp. \propto D^2$ . For solid geometrically similar propellers the weight may be assumed to be proportional to  $D^3$ ; therefore, weight  $\propto (hp.)^{3/2}$ . From the propeller-weight, as well as the diameter, consideration, the engine units should be limited to moderate sizes, which is especially true for high-altitude airplanes because of the pronounced diameter increase with altitude. Since the propeller weight, under the foregoing assumptions, is proportional to  $\left(\frac{1}{\sigma}\right)^{3/2}$ , it can be seen that the 40,000-foot altitude design will be about 8.25 times as heavy as the sea-level design.

It was pointed out in the discussion of figure 27 that increasing the number of blades (increased solidity) slightly reduced the weight and, of course, reduced the efficiency as well (for constant tip speed and air speed). The amount of efficiency that should be sacrificed in order to save weight is a problem that is beyond the scope of the present paper.

#### CONCLUDING REMARKS

The probable order of increasing merit of engine-propeller combinations is: (a) fixed pitch, (b) fixed pitch and variable gear ratio, (c) controllable pitch, (d) controllable pitch and diameter, and (e) controllable pitch

with supercharged engine of optimum supercharging and over-speed capacity.

If the design basic pitch setting of a low-pitch or a moderate-pitch controllable propeller for a given airplane is increased up to about  $35^{\circ}$  or  $40^{\circ}$ , the propeller speed reduced, and the diameter increased, the resulting take-off and high-speed efficiency will be substantially higher.

For light airplanes the cost and weight of controllable propellers might be prohibitive. For such cases fixed-pitch propellers with engines equipped with two-speed gear changes would be preferable if increased take-off thrust were necessary.

For cases wherein the pitch setting is necessarily low because of the inflexibility of the engine or airplane design relative to propeller speeds and diameter, there is little to be gained in performance from the use of special devices such as controllable propellers. The advantage of controllable propellers increases with the design basic pitch setting or roughly, but not necessarily, with the airplane speed.

Controllable propellers, and also variable gear ratios, are more beneficial with supercharged engines than with unsupercharged engines because of several factors, among which is the greater increase in take-off engine power accompanying the increased engine speed. With fixed-pitch propellers the drop in engine speed at take-off is more pronounced for supercharged than for unsupercharged engines unless the power of supercharged engines is considerably increased by boosting.

Further increases in propulsive efficiency for take-off may be obtained at the expense of a small loss in high-speed efficiency by reducing the design pitch setting and increasing the diameter. This system of compromise is very effective for controllable propellers but less effective for fixed-pitch propellers with or without variable gear ratio. The amount of improvement possible depends upon the design basic pitch; the higher the pitch, the greater the possible improvement.

Several factors associated with supercharged engines influence the amount of compromise desirable for controllable-propeller design. The fact that the propellers are

designed for critical altitude flight and overspeeding the engine at take-off necessitates reducing the blade angle, with the same effect on the take-off thrust as compromising; but boosting the engine at take-off has the reverse effect. Since overspeeding and boosting both increase the brake horsepower, overspeeding is preferable when the maximum power is limited.

Controllable propellers designed for high-altitude flight with supercharged engines will have excellent take-off characteristics because of the large diameter and low pitch necessitated by sea-level operation.

There is no well-defined limiting value of speed, power, or altitude in future propeller designs, but increasing each factor imposes increased design difficulties. Higher speeds are accompanied by higher pitches, greater powers necessitate greater total blade or disk area, and higher altitudes require larger diameters. Designers must be prepared for pitch settings of  $45^{\circ}$  or  $50^{\circ}$  for air speeds of about 400 miles per hour. The diameters of 3-blade propellers may reach 15 feet for 3,000-horsepower engines for sea-level altitude; the diameter must, however, be about doubled if the altitude is increased to 40,000 feet, assuming constant air speed and tip speed. The diameter may be confined to practicable limits for high-altitude airplanes if the engines are limited to low powers.

Langley Memorial Aeronautical Laboratory,  
National Advisory Committee for Aeronautics,  
Langley Field, Va., May 15, 1936.

## APPENDIX

Since the main paper was written, there have been made available additional full-scale propeller data that may be of use to the designer. The test results from which this information was obtained are given in reference 7, together with the nacelle outline drawing. A photograph of the test set-up is shown in figure 28. Figure 29 is a composite photograph of the propellers referred to in the reference as B, C, D. and E. The respective drawing numbers are: Hamilton Standard 1C1-0, three blades; Navy plan form 5868-9, three blades; Navy plan form 5868-9, two blades; and Navy plan form 3790, three blades. The blade form curves for propellers B, C, D, and E, together with those for propeller 4412, are given in figure 30. Propeller E and propeller 4412 are designed with the R.A.F. 6 as the basic section; the other propellers are designed with the Clark Y section. Propeller B was tested with two pitch distributions: the normal pitch distribution and with the pitch washed out at the tip, the latter denoted as  $B_x$  or 1C1-0X. This change in pitch distribution is also shown in figure 30. Speed-power-coefficient  $C_s$  charts for propellers B,  $B_x$ , C, D, and E are given in figures 31 to 35. Figure 36 shows the ratio of the power absorbed by 2- and 3-blade propellers (5868-9). It may be noted that the power absorbed by the 2-blade propeller relative to the 3-blade one decreases uniformly from about 0.74 for the  $15^\circ$  pitch setting to about 0.68 for the  $45^\circ$  blade angle. The mean of these values closely checks that assumed for the construction of section D, figure 2. The relative efficiencies of 2- and 3-blade propellers are given in figure 37 for four flight conditions and over a wide range of  $C_s$  values. The 2-blade propeller has about 2.5 percent higher efficiency at the high-speed or cruising condition and about 1.5 percent higher value at the climb, but there seems to be little advantage in either over the other at the take-off condition.

Figure 38 is a comparison of propellers 1C1-0 and 1C1-0X. The washed-out pitch distribution decreases the efficiency at the high-speed, cruising, and climb conditions for low  $C_s$  or pitch values but increases the efficiency for the same conditions at high  $C_s$  values. The take-off efficiency suffers over the whole range because of this washout feature.

It has been suggested that sections A of figure 2 be

made up for the propellers included in this appendix; but, since this would involve a considerable addition, it was thought that a more detailed explanation of the construction of this section of the figure with an example would suffice. Persons interested in any propeller can readily make up section A by following the method outlined as follows:

Example 5.- Construction of section A for the 3-blade propeller 5868-9.

(1) The chart, figure 2, was constructed for propeller 4412, which has a total blade-width ratio of 0.105 at  $3/4 R$ ; consequently, if a section A for a different blade-width propeller is to be used in conjunction with the chart; the power should be corrected. The total blade-width ratio of the 3-blade propeller 5868-9 is 0.1845. Reference to section D of figure 2 shows that the latter propeller absorbs 1.6 times the power of the 4412 propeller; therefore, the power must be corrected by this factor in order that section A for propeller 5868-9 can be used directly in conjunction with the rest of figure 2.

$$C_s = \frac{0.638 \times \text{m.p.h.} \times \sigma^{1/5}}{(1.6 \text{ hp.})^{1/5} \times \text{r.p.m.}^{2/5}} \quad (\text{a})$$

or

$$1.096 C_s = \frac{0.638 \times \text{m.p.h.} \times \sigma^{1/5}}{\text{hp.}^{1/5} \times \text{r.p.m.}^{2/5}}$$

also

$$C_p^{1/5} = \frac{V/nD}{1.096 C_s} \quad (\text{b})$$

(2) Since the lines for constant  $C_p^{1/5}$ , pitch setting and  $\eta$  are straight and pass through zero, it is necessary to solve for only one point on each line. Therefore, let  $\text{hp.}^{1/5} \times \text{r.p.m.}^{2/5} = 90$ . From (a)

$$\text{m.p.h.} \times \sigma^{1/5} = 154.8 C_s \quad (\text{c})$$

## (3) Tabulating the observed and derived data:

24

From figure 33 (envelope)					Computed				
Pitch setting	$C_s$	$\frac{V}{nD}$	$\eta$	$C_p^{1/5}$	m.p.h. $\times \sigma^{1/5}$ for $hp^{1/5} r.p.m^{2/5} = 90$	Sea-level air speed for tip speeds of -			
						800 f.p.s.	900 f.p.s.	1,000 f.p.s.	1,100 f.p.s.
Degrees						m.p.h.	m.p.h.	m.p.h.	m.p.h.
15	1.16	0.60	0.793	0.472	180	102	115	127	140
20	1.49	.82	.832	.501	231	137	154	171	189
25	1.80	1.05	.850	.531	279	171	194	216	238
30	2.12	1.30	.850	.558	328	207	235	261	288
35	2.45	1.57	.850	.583	380	242	274	307	334
40	2.80	1.87	.840	.605	435	279	315	348	383
45	3.21	2.24	.825	.633	497	318	354	394	437

The observed data in the table are read directly from figure 33. The value of  $C_p^{1/5}$  is then computed by using the relation (b), and m.p.h.  $\times \sigma^{1/5}$  is determined from the relation (c). Values of m.p.h. for different tip speeds are read from figure 39.

(4) In order to plot whole values of  $C_P^{1/5}$ ,  $\eta$ , and pitch setting in the chart, these quantities from the foregoing table are plotted against  $m.p.h. \times \sigma^{1/5}$  and the whole values determined (plot not shown). In figure 40  $C_P^{1/5}$  lines are drawn in (dotted) through the origin and the points established at  $hp.^{1/5} r.p.m.^{2/5} = 90$ . The abbreviated pitch-setting lines (solid) are also similarly drawn in. The lines of constant tip speed may be plotted with reference to m.p.h. at sea level and to the  $C_P^{1/5}$  lines. The  $\eta$  lines may then be drawn straight to a tip-speed value of about 980 feet per second. Since the  $\eta$  decreases roughly 10 percent for an increase in tip speed from 1,000 to 1,100 feet per second, the  $\eta$  curves may be corrected accordingly. Since operation above 1,000 feet per second is of only slight interest, the accuracy of that portion of the chart is of little importance.

This chart may be used directly in conjunction with figure 2 with one modification. Since the test results are for the 3-blade propeller, the origin of the solidity-efficiency-correction curve (section G) should be shifted to the blade-width ratio of 0.1845.

Had the power not been corrected for differences in blade width, the derived section A could be used in conjunction with figure 2 if the horsepower were read from section D at a value of blade-width ratio of 0.105. This condition would limit the application of the chart to the particular propeller in question.

## REFERENCES

1. Windler, Ray: Tests of a Wing-Nacelle-Propeller Combination at Several Pitch Settings up to 42°. T.R. No. 564, N.A.C.A., 1936.
2. Wood, Donald H.: Full-Scale Tests of Metal Propellers at High Tip Speeds. T.R. No. 375, N.A.C.A., 1931.
3. Weick, Fred E.: Aircraft Propeller Design. McGraw-Hill Book Company, Inc., 1930.
4. Russell, J. S., and McCoy, H. M.; Wind Tunnel Tests on a High Wing Monoplane with Running Propeller. Part I. Propulsive Characteristics of Two and Three Bladed Adjustable Pitch Propellers Extended to Fairly Large Blade Angles. Jour. Aero. Sciences, Jan, 1936, pp. 73-78.
5. Lock, C. N. H., Bateman, H., and Nixon, H. L.: Wind Tunnel Tests of High Pitch Airscrews. Part I. R. & M. No. 1673, British A.R.C., 1935.
6. Hartman, Edwin P.: Considerations of the Take-Off Problem. T.N. No. 557, N.A.C.A., 1936.
7. Theodorsen, Theodore, Stickle, George W., and Brevoort, M. J.: Characteristics of Six Propellers including the High-Speed Range. Confidential Memorandum Report, N.A.C.A., 1936.

TABLE I

Summary Table of  $\frac{t.h.p.}{b.h.p.m_c}$  for Various Engine-Propeller Combinations

Flight condition	Engine propeller combination	Type of engine	Unsupercharged engines			Supercharged engine operating at various ratings at sea level							
		Operating condition	Full throttle			High-speed rating		Take-off rating		Full-throttle			
		Pitch-setting for high speed, deg.	22	32	42	22	32	42	22	32	42	22	
Take-off condition (0.3 high speed)	Fixed pitch	0.338	0.342	0.332	0.307	0.315	0.304	0.320	0.325	0.317	0.427	0.435	0.420
		" " , compromise (3° under basic pitch)	.361	.362	.330	-	-	-	-	-	-	-	-
		" " , variable gear ratio	.370	.380	.370	.365	.390	.375	.396	.412	.400	.473	.500
	Controllable pitch (peak $\eta$ at high speed)	.397	.425	.462	.422	.507	.540	.469	.547	.585	.533	.567	.620
		" " , compromise (3° under basic pitch)	.435	-	-	-	.556	.615	-	.615	.675	-	.690
		" " " (5° under basic pitch)	-	.565	.615	-	-	-	-	-	-	-	-
		" " , 1.25 to 1.00 over-speed drive and diameter	-	.565	.625	-	-	-	-	-	-	-	-
		" " and diameter	.452	.607	.715	.430	.582	.675	.471	.634	.740	.592	.815
	Fixed pitch	.655	.657	.625	.570	.575	.538	.605	.611	.574	.827	.837	.790
		" " , compromise (3° under basic pitch)	.680	.680	.627	-	-	-	-	-	-	-	-
		" " , variable gear ratio	.702	.735	.700	.673	.715	.682	.732	.777	.743	.925	.965
	Controllable pitch (peak $\eta$ at high speed)	.700	.770	.788	.680	.750	.760	.745	.825	.850	.920	1.018	1.063
		" " , compromise (3° under basic pitch)	.715	-	-	-	.760	.771	-	.840	.855	-	1.077
		" " " (5° under basic pitch)	-	.802	.842	-	-	-	-	-	-	-	-
		" " , 1.25 to 1.00 over-speed drive and diameter	-	.815	.835	-	-	-	-	-	-	-	-
		" " and diameter	.725	.822	.842	.682	.767	.771	.745	.842	.860	.960	1.090
Climb (0.65 high speed)	Fixed pitch	.822	.837	.775	-	-	-	-	-	-	1.045	1.057	.988
		" " , compromise (3° under basic pitch)	.815	.830	.757	-	-	-	-	-	-	-	-
		" " , variable gear ratio	.822	.837	.775	-	-	-	-	-	1.090	1.110	1.045
	Controllable pitch (peak $\eta$ at high speed)	.822	.837	.775	-	-	-	-	-	-	1.087	1.110	1.045
		" " , compromise (3° under basic pitch)	.815	-	-	-	-	-	-	-	-	1.100	1.020
		" " " (5° under basic pitch)	-	.807	.737	-	-	-	-	-	-	-	-
		" " , 1.25 to 1.00 over-speed drive and diameter	-	.817	.745	-	-	-	-	-	-	-	-
		" " and diameter	.822	.837	.775	-	-	-	-	-	1.090	1.110	1.045

TABLE II

Propeller-Design Trend with Increasing Values of Air Speed,

Altitude, and Power

(3-blade, 4412 propeller, 1,000-foot-per-second tip speed;  
 $r.p.m. \propto \sigma^{1/2}$ , diameter  $\propto hp^{1/2}$ , and  $r.p.m. \times \text{diameter} = K$  for constant values of air speed or pitch setting.)

Altitude	200 miles per hour			300 miles per hour			400 miles per hour		
	Diameter	r.p.m.	Pitch setting	Diameter	r.p.m.	Pitch setting	Diameter	r.p.m.	Pitch setting
750 horsepower									
Feet	Feet		deg.	Feet		deg.	Feet		deg.
0	8.3	2,200	26.9	7.4	2,320	36.2	6.7	2,310	48.0
10,000	9.6	1,900	26.9	8.6	1,990	36.2	7.8	1,985	48.0
20,000	11.4	1,600	26.9	10.1	1,690	36.2	9.2	1,690	48.0
30,000	13.5	1,350	26.9	12.1	1,420	36.2	11.0	1,410	48.0
40,000	16.7	1,090	26.9	14.9	1,150	36.2	13.6	1,140	48.0
1,500 horsepower									
0	11.7	1,560	26.9	10.5	1,630	36.2	9.5	1,630	48.0
10,000	13.6	1,340	26.9	12.2	1,410	36.2	11.1	1,390	48.0
20,000	16.1	1,130	26.9	14.3	1,200	36.2	13.0	1,190	48.0
30,000	19.1	960	26.9	17.1	1,000	36.2	15.6	990	48.0
40,000	23.6	775	26.9	21.1	815	36.2	19.2	805	48.0
2,250 horsepower									
0	14.4	1,270	26.9	12.8	1,360	36.2	11.6	1,335	48.0
10,000	16.6	1,100	26.9	14.9	1,150	36.2	13.5	1,145	48.0
20,000	19.7	925	26.9	17.5	975	36.2	15.9	975	48.0
30,000	23.4	780	26.9	21.0	820	36.2	19.1	815	48.0
40,000	29.0	630	26.9	25.8	665	36.2	23.6	660	48.0

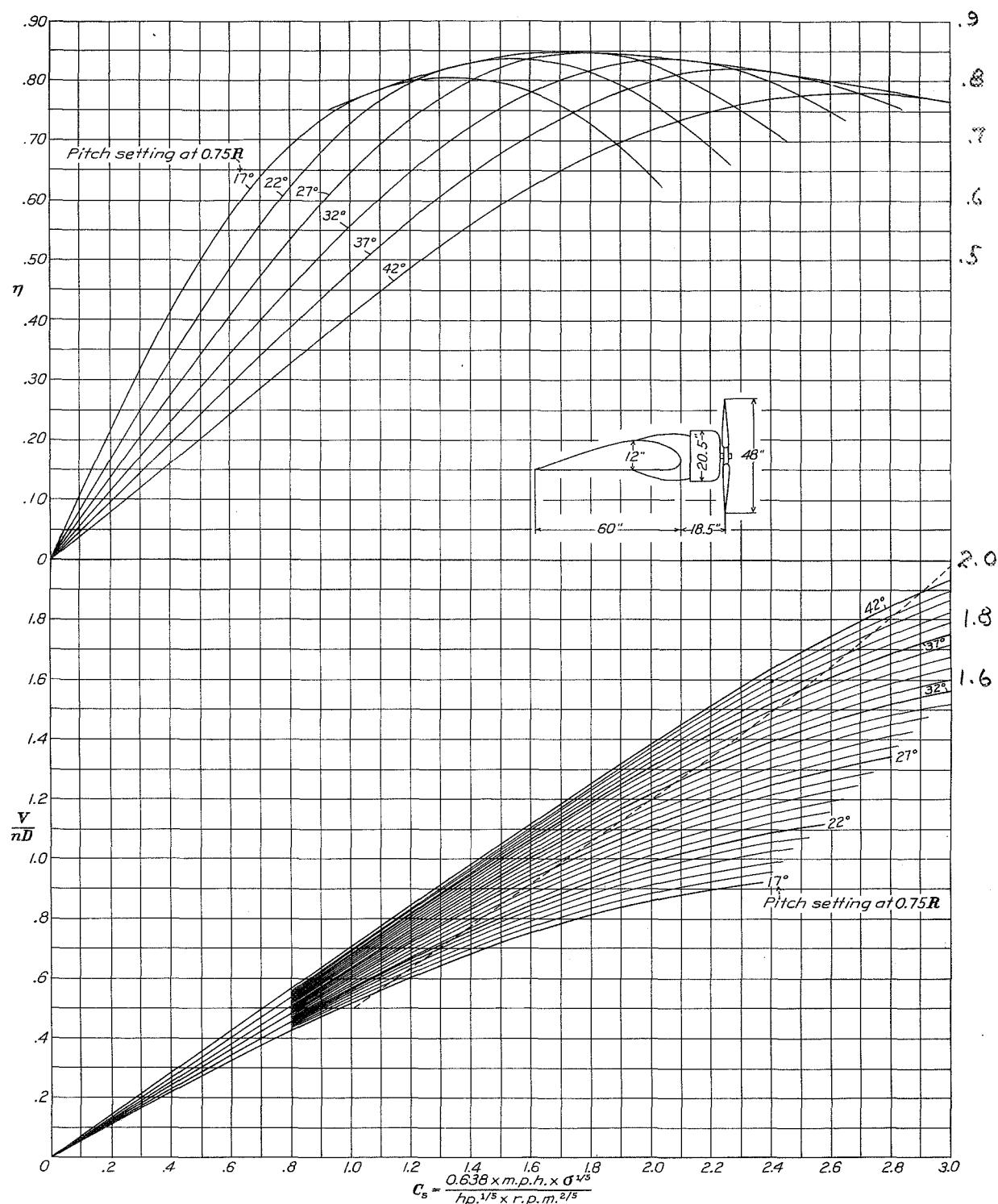


Figure 1.- Two-blade propeller characteristics; N.A.C.A. cowled nacelle; 4412 propeller.

Fig. 2  
Sections A, B.  
Figure contin-  
ued on follow-  
ing pages.

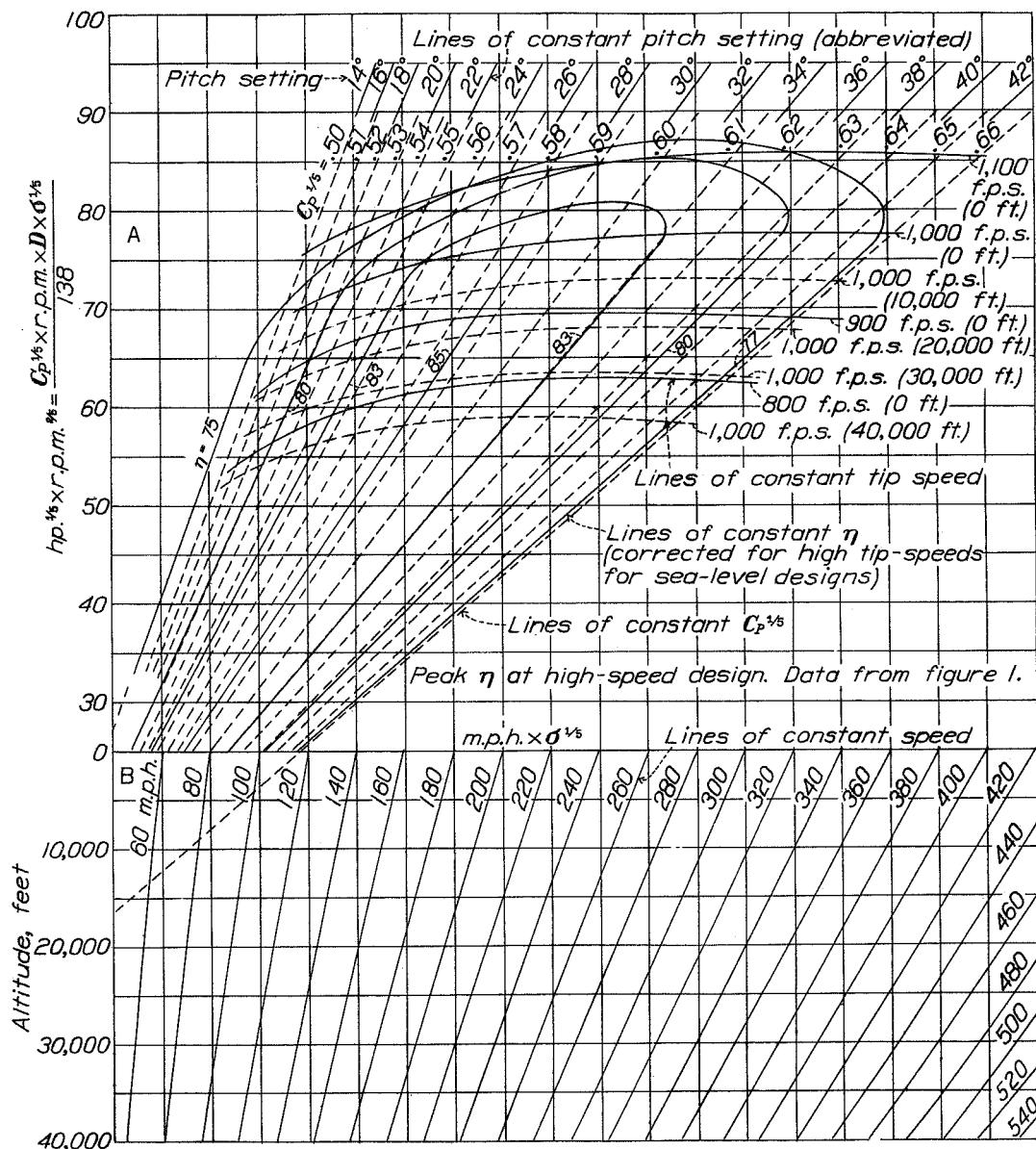
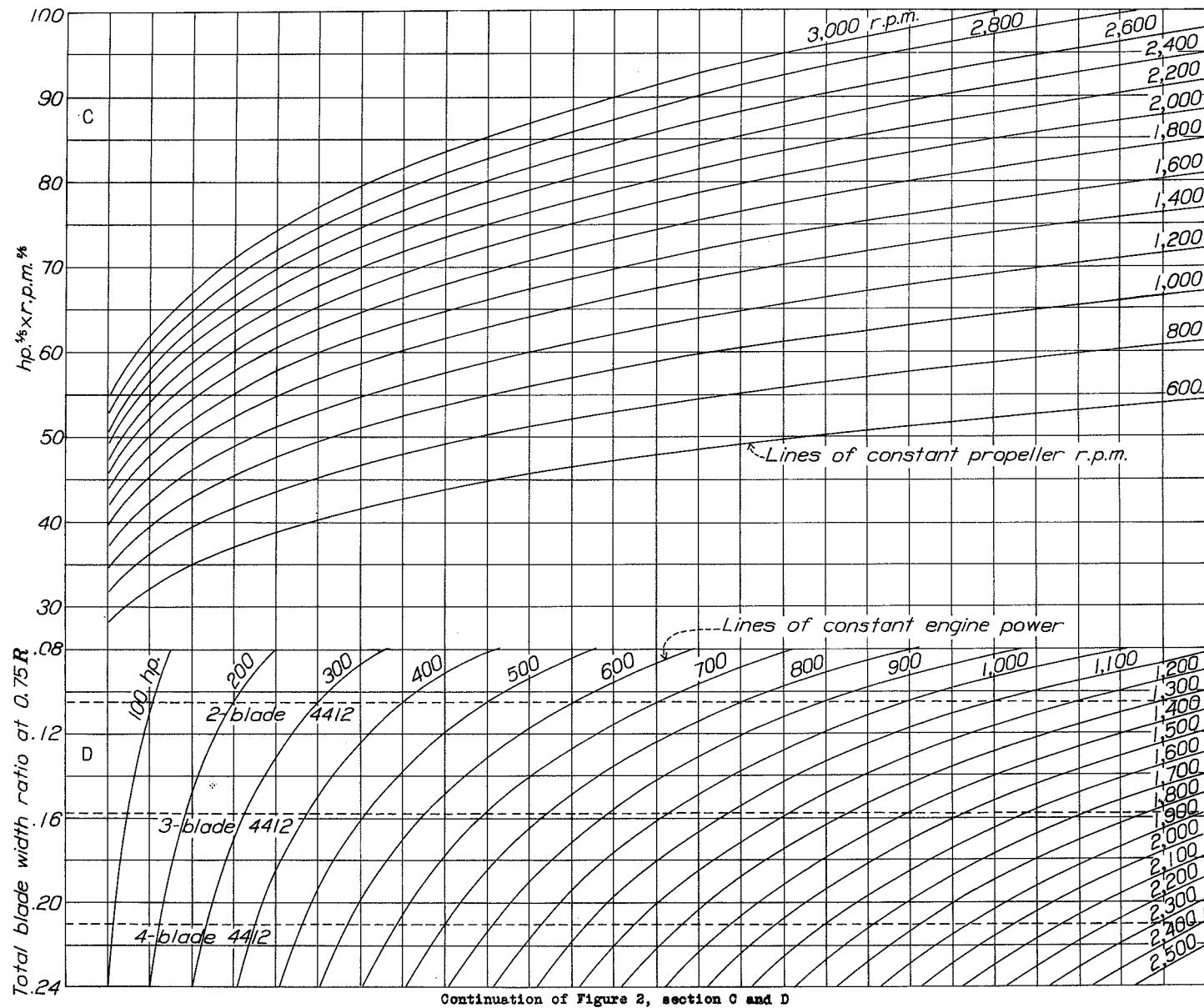
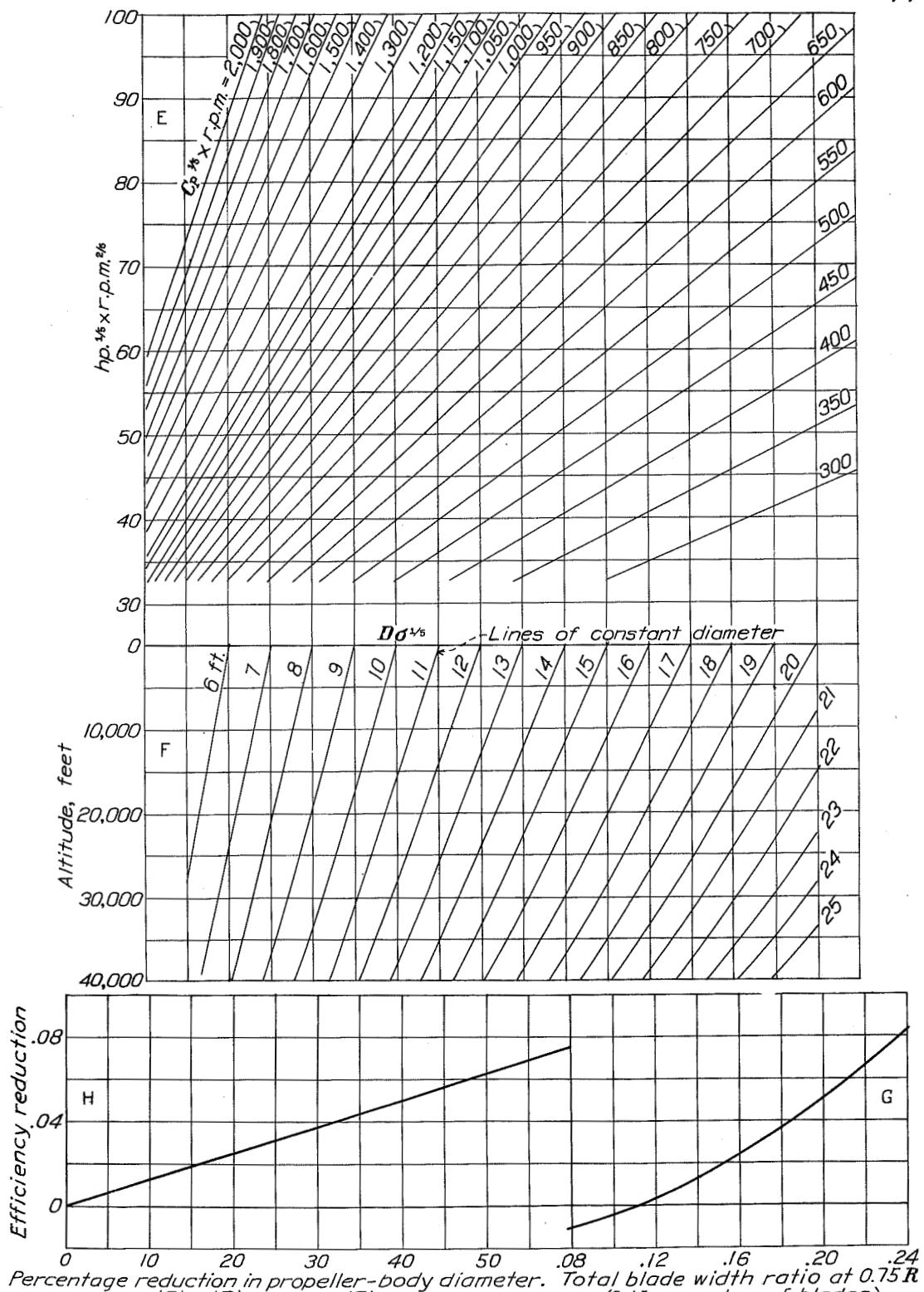


Figure 2.—Chart correlating experimental propeller test data with various design parameters.  
Section A gives the necessary experimental data while sections B, C, D, E and F give corresponding design dimensional data. Sections A and H give  $\eta$  correction for solidity and propeller-body interference. Read from section to section horizontally or vertically  
In order to obtain the diameter, first solve for  $C_p^{1/5} \times r.p.m.$   
(Continued on following pages.)



N.A.C.A.

FIG. 3  
Sections C, D



Continuation of Figure 2, sections E, F, G, and H.

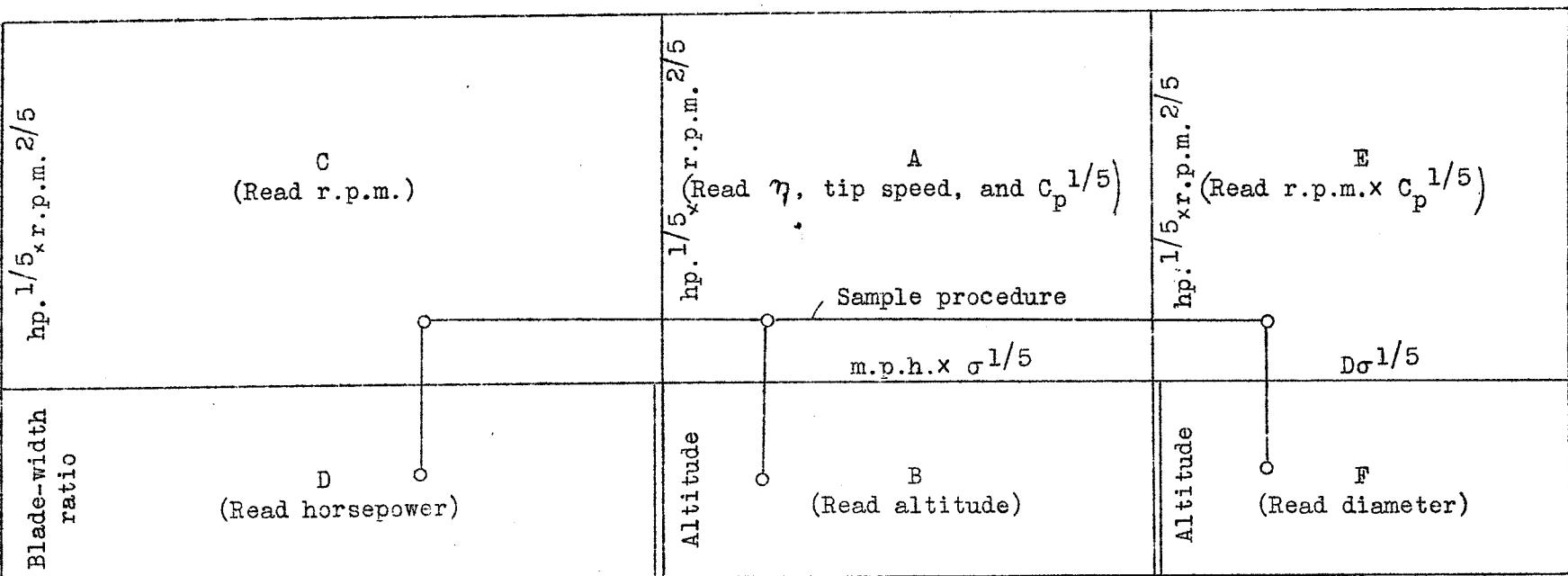
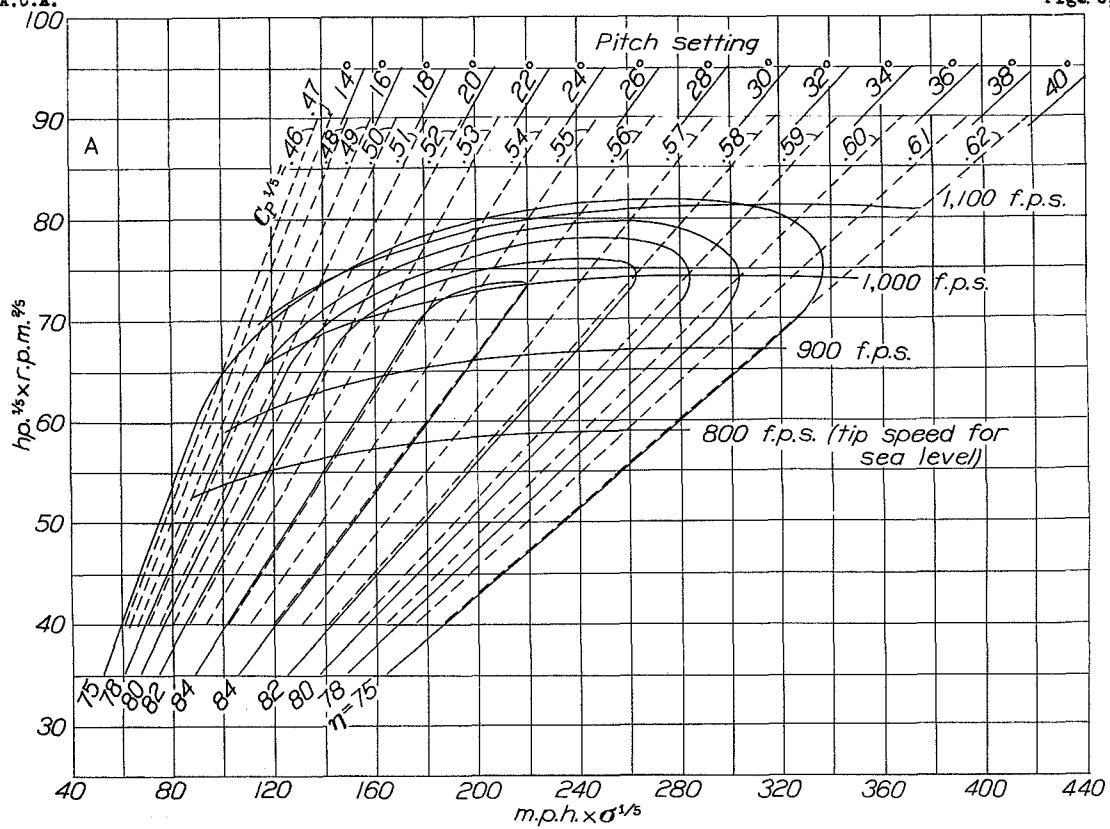
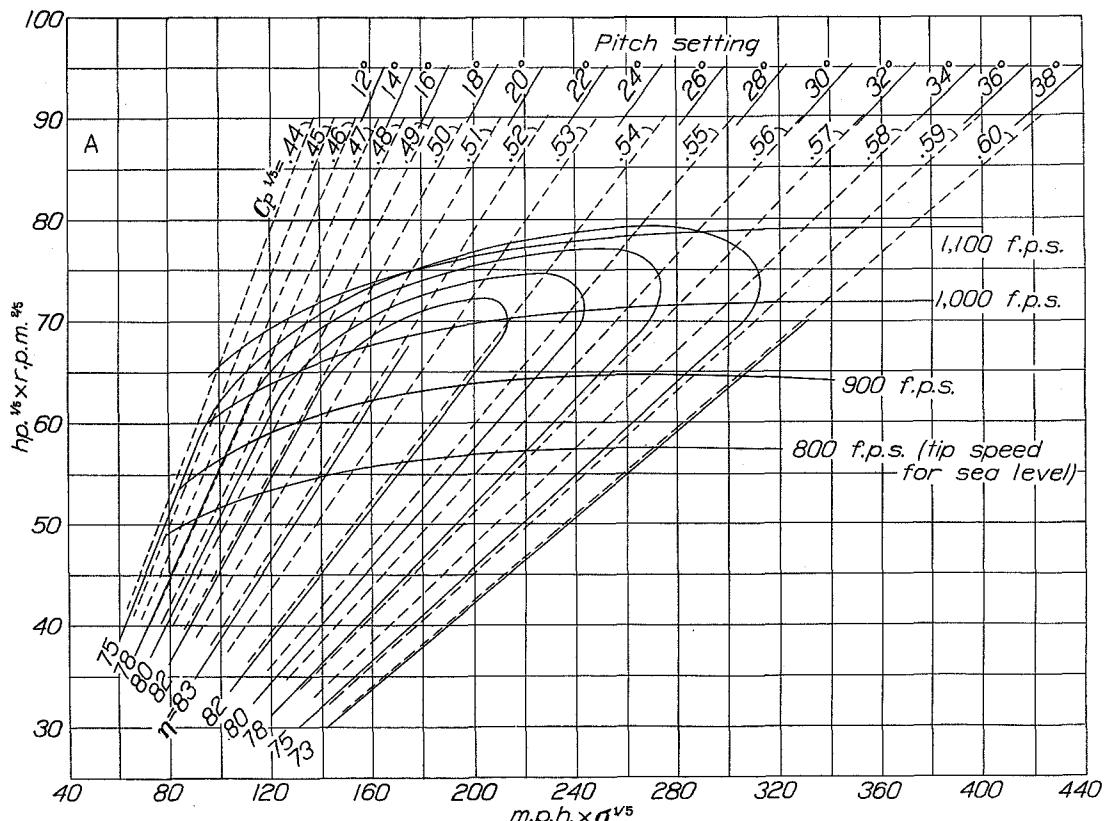


Figure 2a.- Diagram illustrating the use of figure 2.

Figure 3.—Compromise propeller; 3° under  $\eta_{\max}$  envelope. Data taken from figure 1.Figure 4.—Compromise propeller; 5° under  $\eta_{\max}$  envelope. Data taken from figure 1.

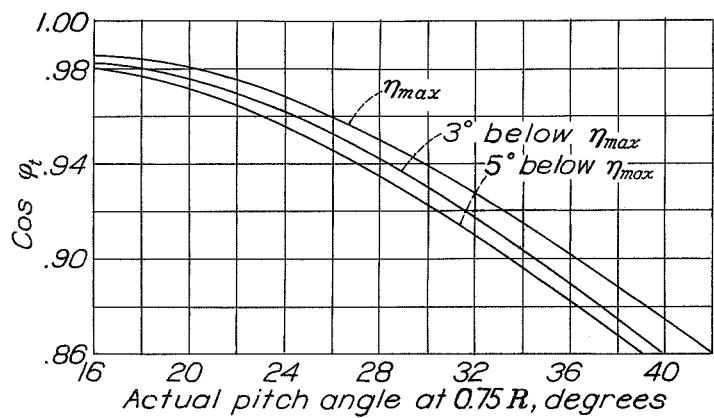


Figure 5.-Curves for finding  $\cos \varphi_t$ . Tip speed =  $\frac{\pi \times r.p.m. \times D}{60 \cos \varphi_t}$

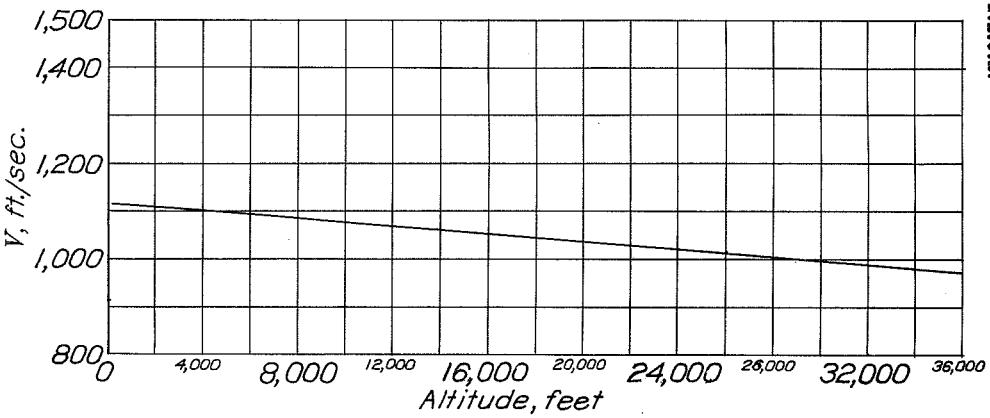


Figure 6.-Velocity of sound in standard atmosphere.

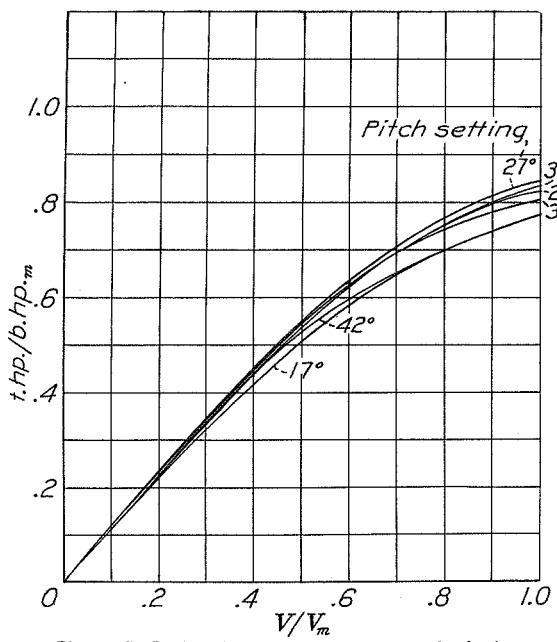


Figure 7.-Ratio of thrust horsepower to brake horsepower available at high speed for fixed-pitch propellers designed with maximum efficiency at high speed. Unsupercharged engines.

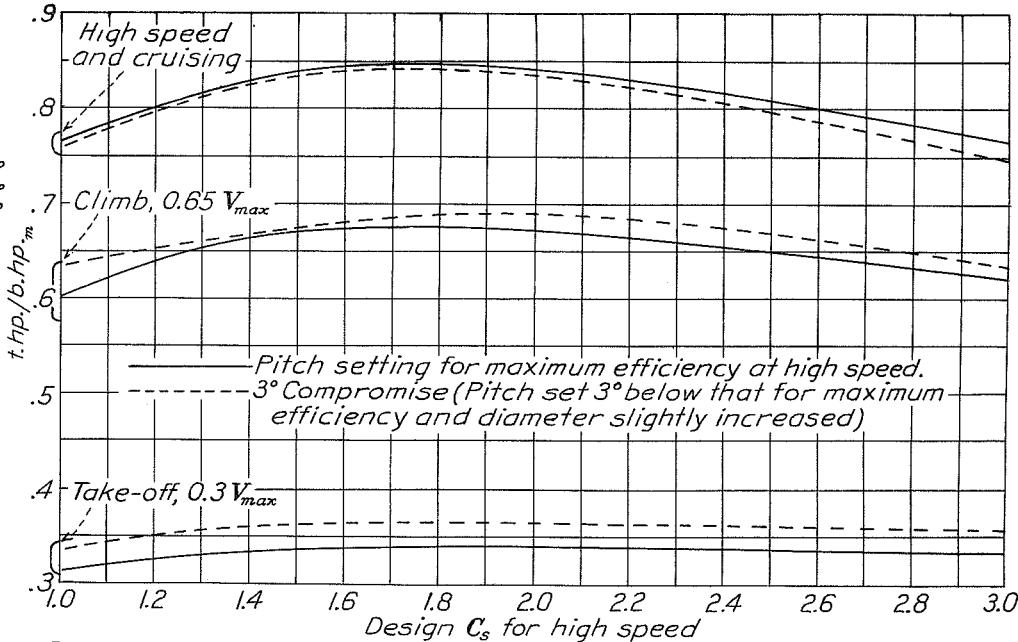


Figure 8.-Comparison between high-speed and compromise design propellers. Unsupercharged engines and fixed-pitch propellers.

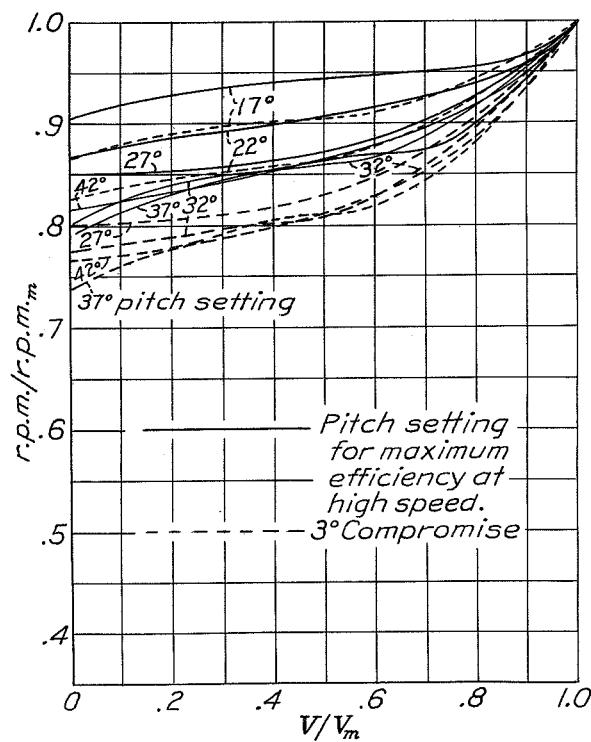


Figure 9.—Change in full-throttle engine speed with changes in air speed. Unsupercharged engines and fixed-pitch propellers.

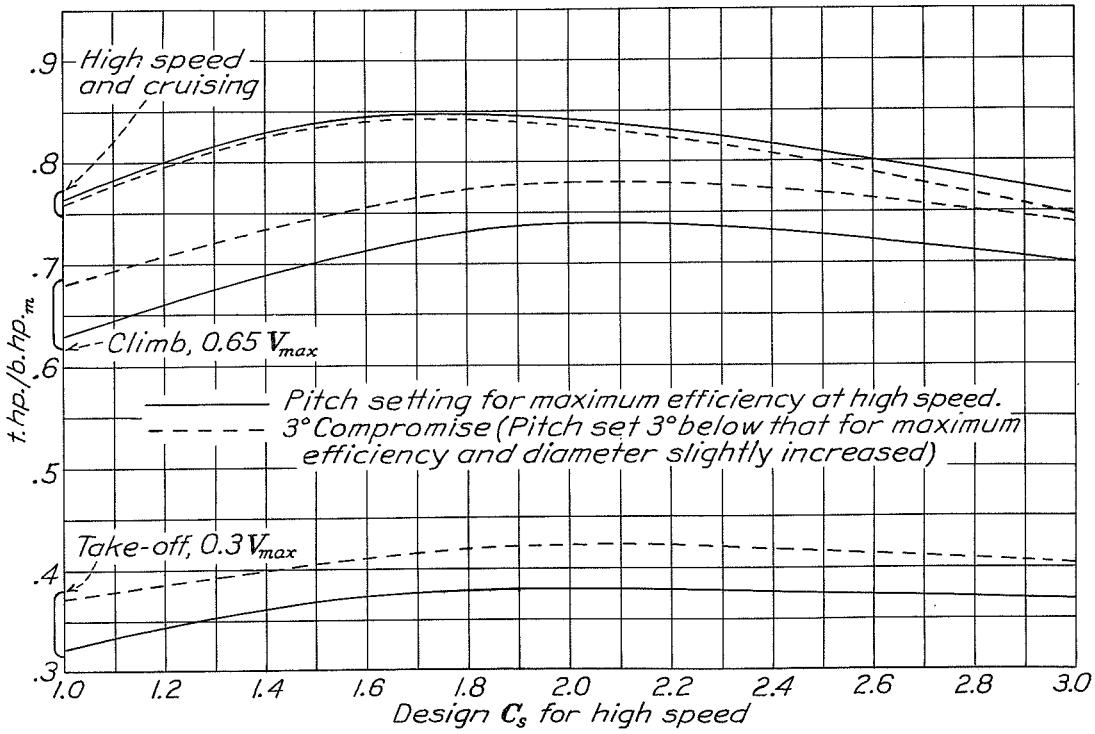


Figure 10.—Comparison of high-speed and compromise design propellers. Unsupercharged engines with variable gear ratio and fixed-pitch propellers.

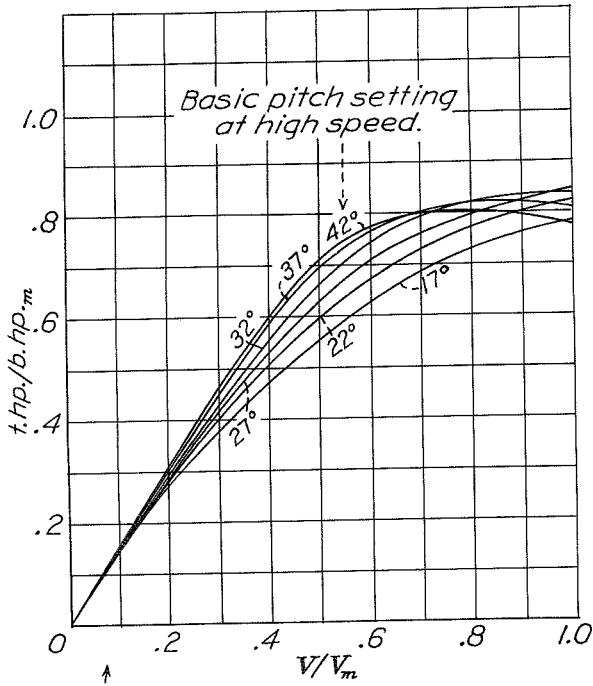


Figure 11.- Ratio of thrust horsepower to brake horsepower available at high speed for controllable propellers designed with maximum efficiency at high speed. Unsupercharged engines.

Figure 13.- Controllable-pitch propellers having various degrees of compromise and also controllable pitch and diameter. Unsupercharged engines.

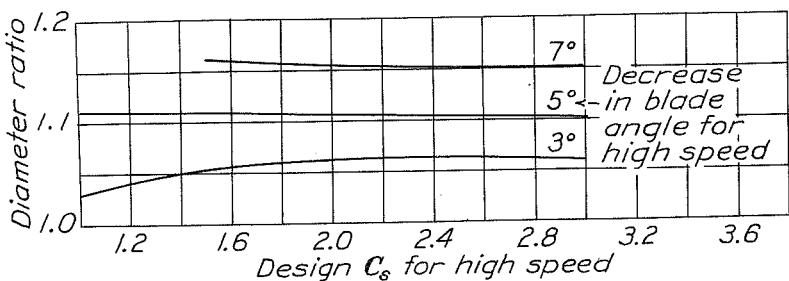


Figure 14.-Changes in propeller diameter accompanying various amounts of design blade-angle reduction for compromise designs.

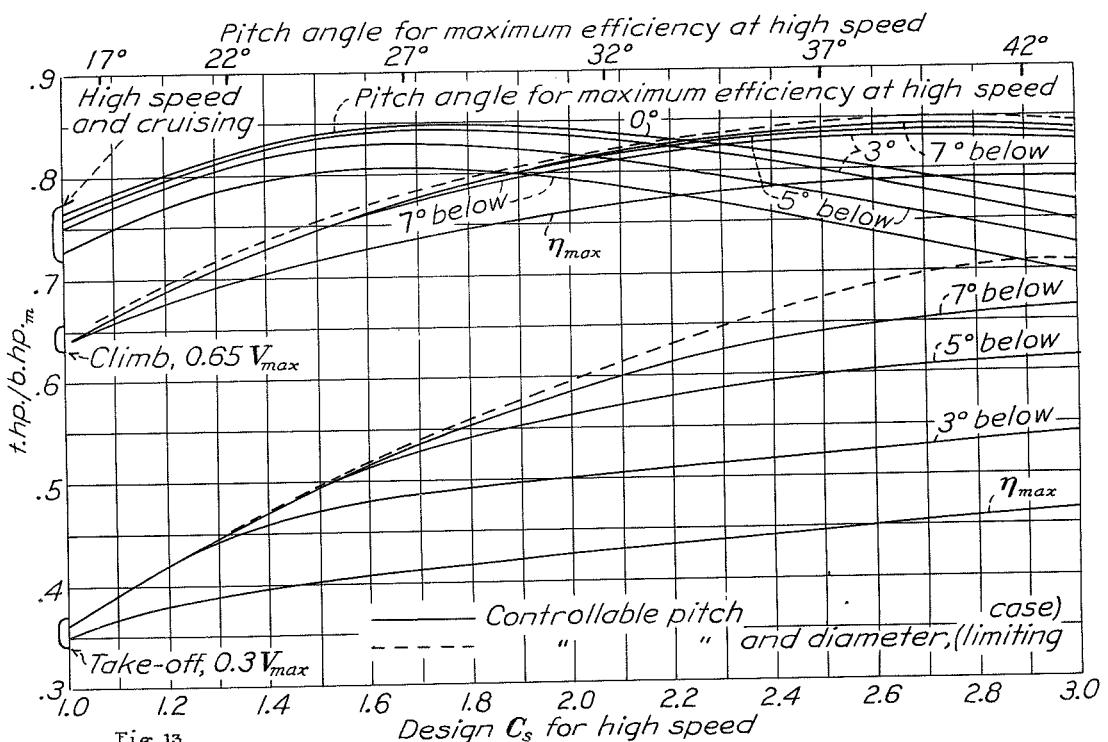


Fig. 13

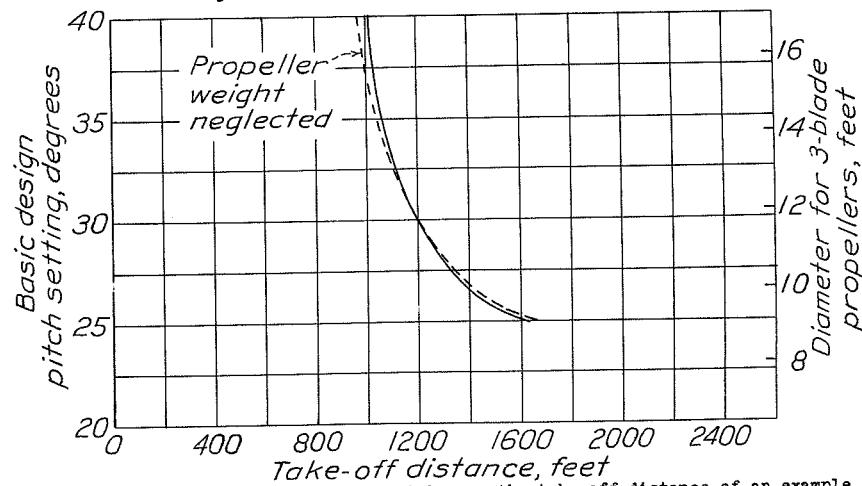


Figure 15.-Effect of design pitch setting on the take-off distance of an example airplane. (Controllable propellers, maximum compromise.)

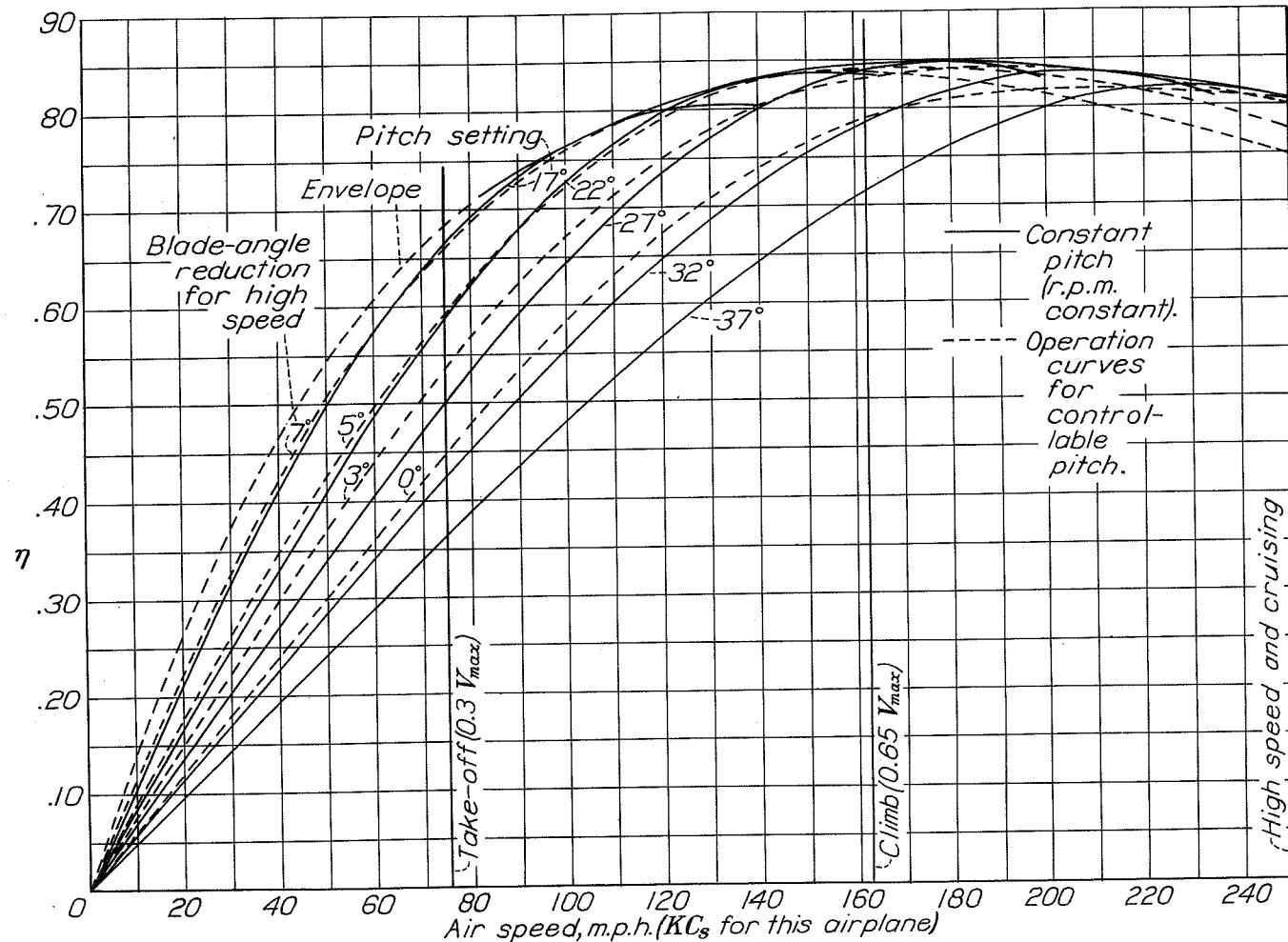


Figure 12.—Operation-efficiency curves of various compromise controllable propellers from take-off to high speed of an airplane chosen as an example.

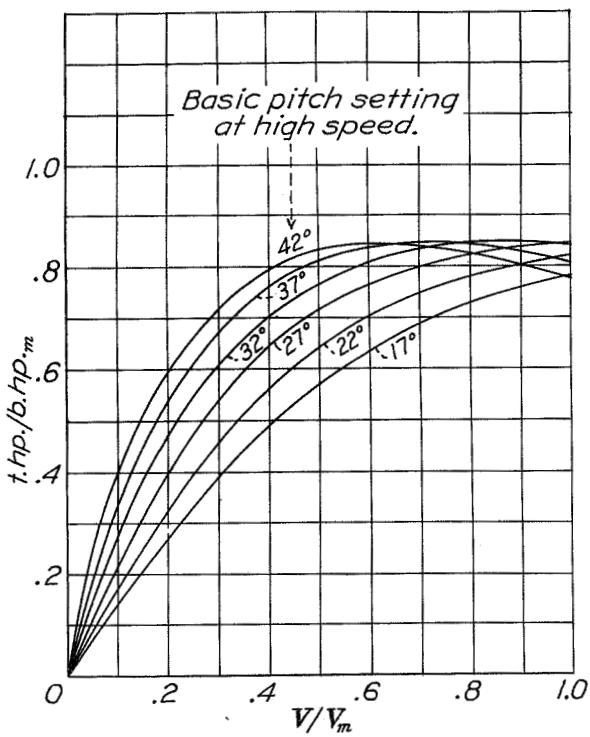


Figure 16.—Ratio of thrust horsepower to brake horsepower available at high speed for controllable pitch and diameter propellers. Unsupercharged engines.

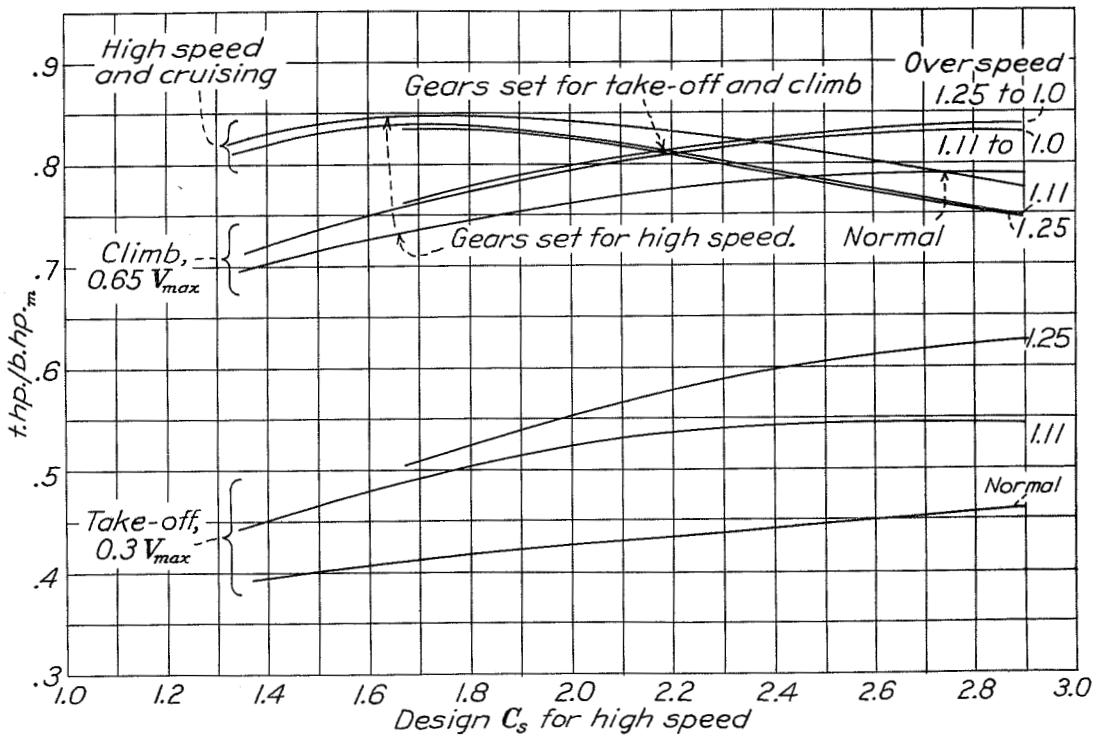


Figure 18.—Controllable propellers with one or two engine gear changes. Unsupercharged engines.

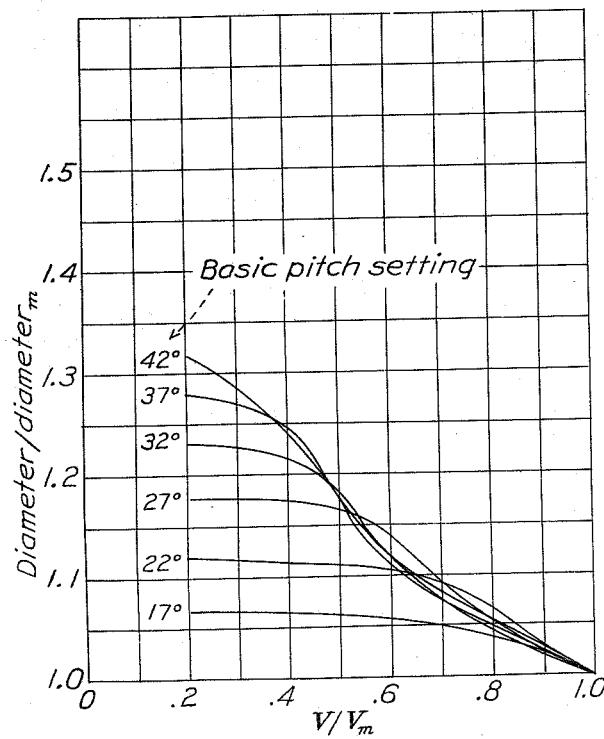


Figure 17.—Change in diameter with change in air speed for controllable pitch and diameter propellers.

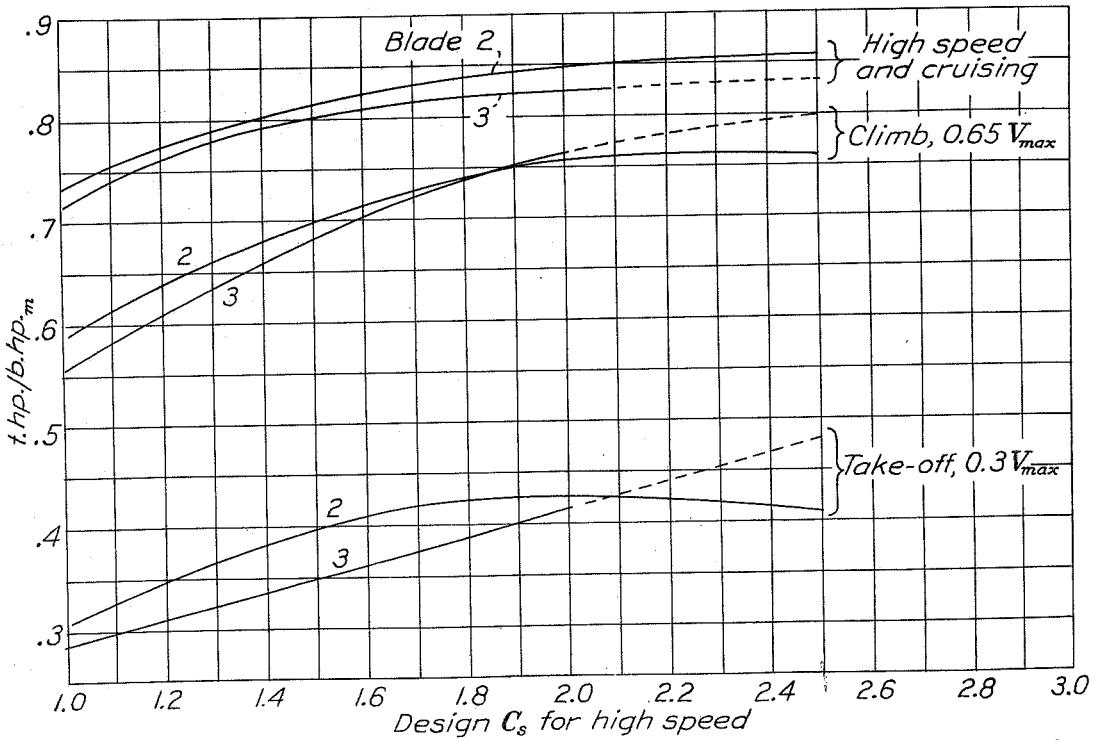


Figure 18.—Comparison between 2 and 3-blade propellers. Data taken from reference 4. Fixed-pitch propellers and unsupercharged engines.

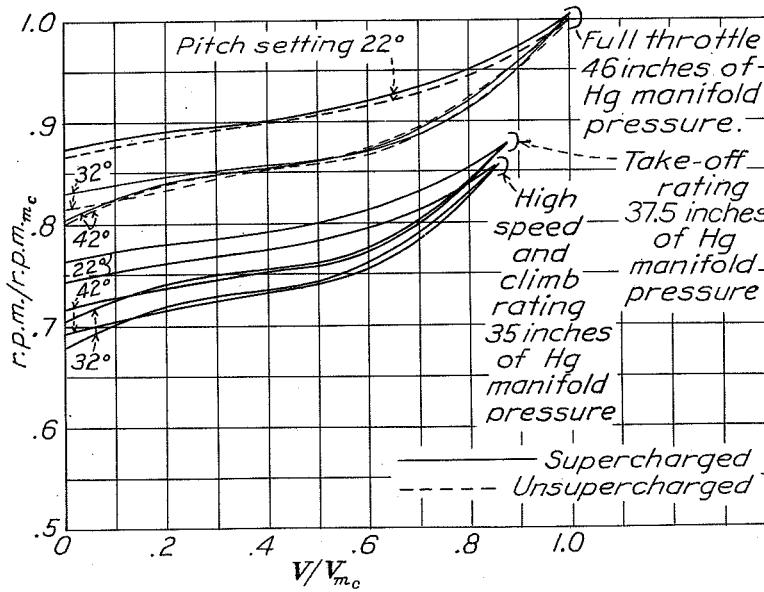
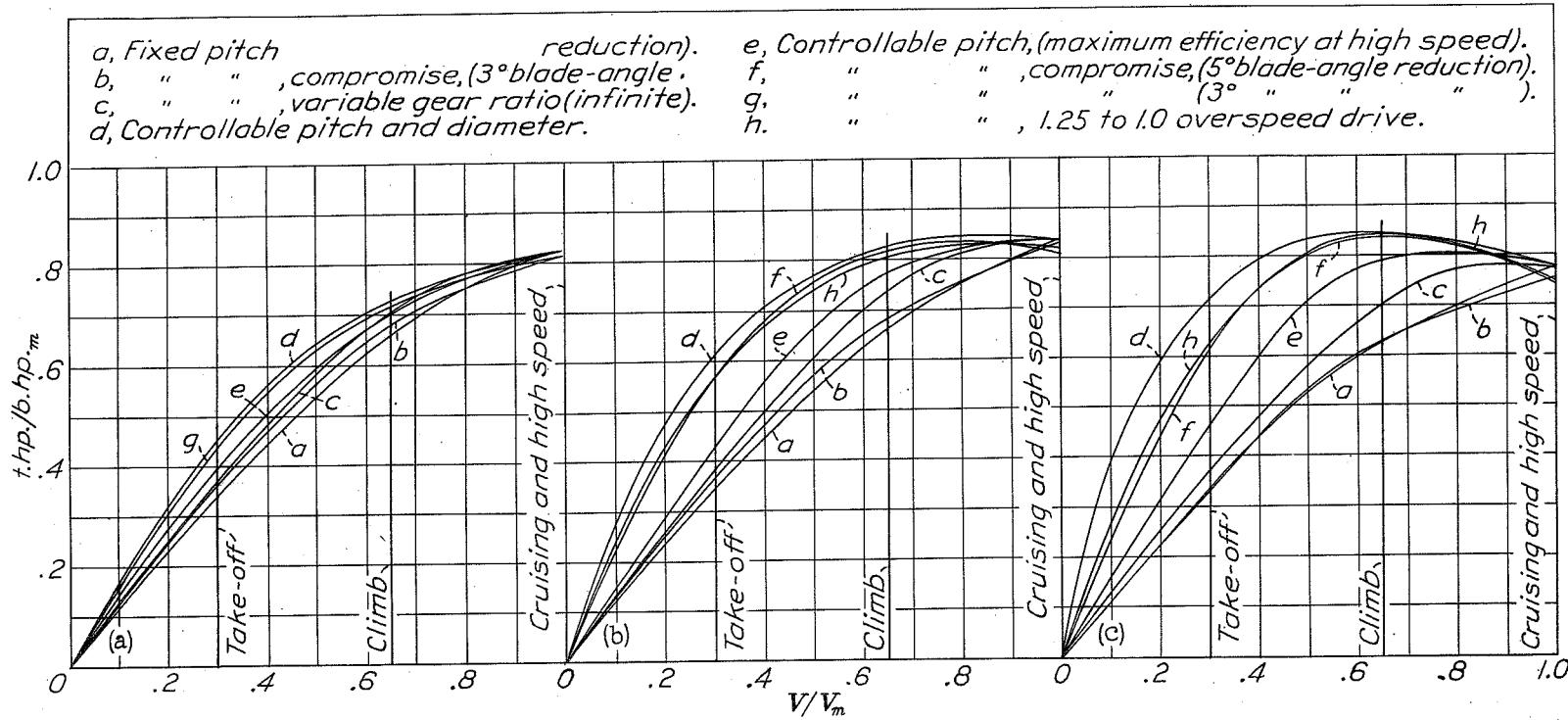


Figure 20.—Comparison between various engine-propeller combinations for various pitch settings. Unsupercharged engines.

Figure 21.—Change in engine speed with air speed for sea-level operation of supercharged and unsupercharged engines with fixed-pitch propellers. (Critical altitude, 7,000 feet.)  
 Miss. 30, 21

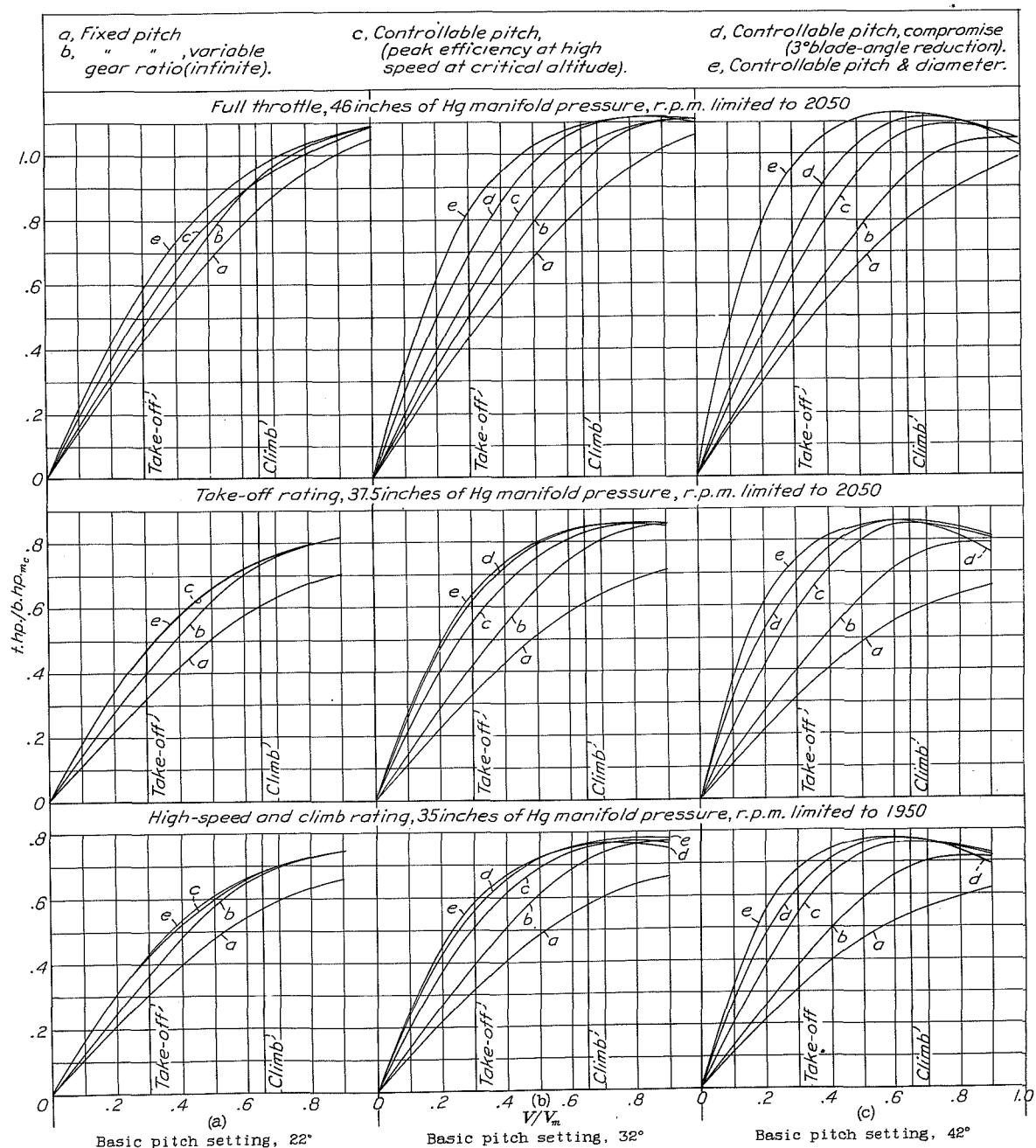
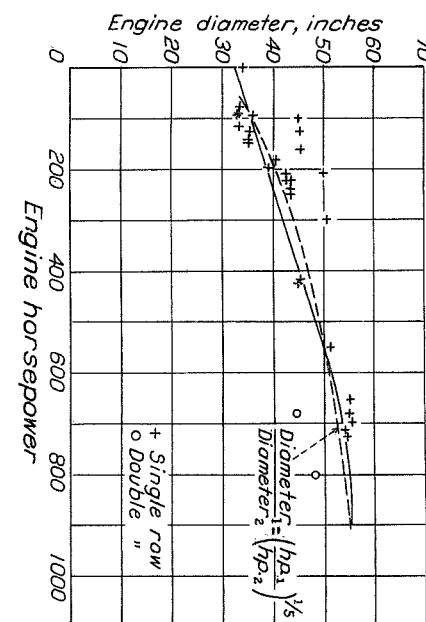
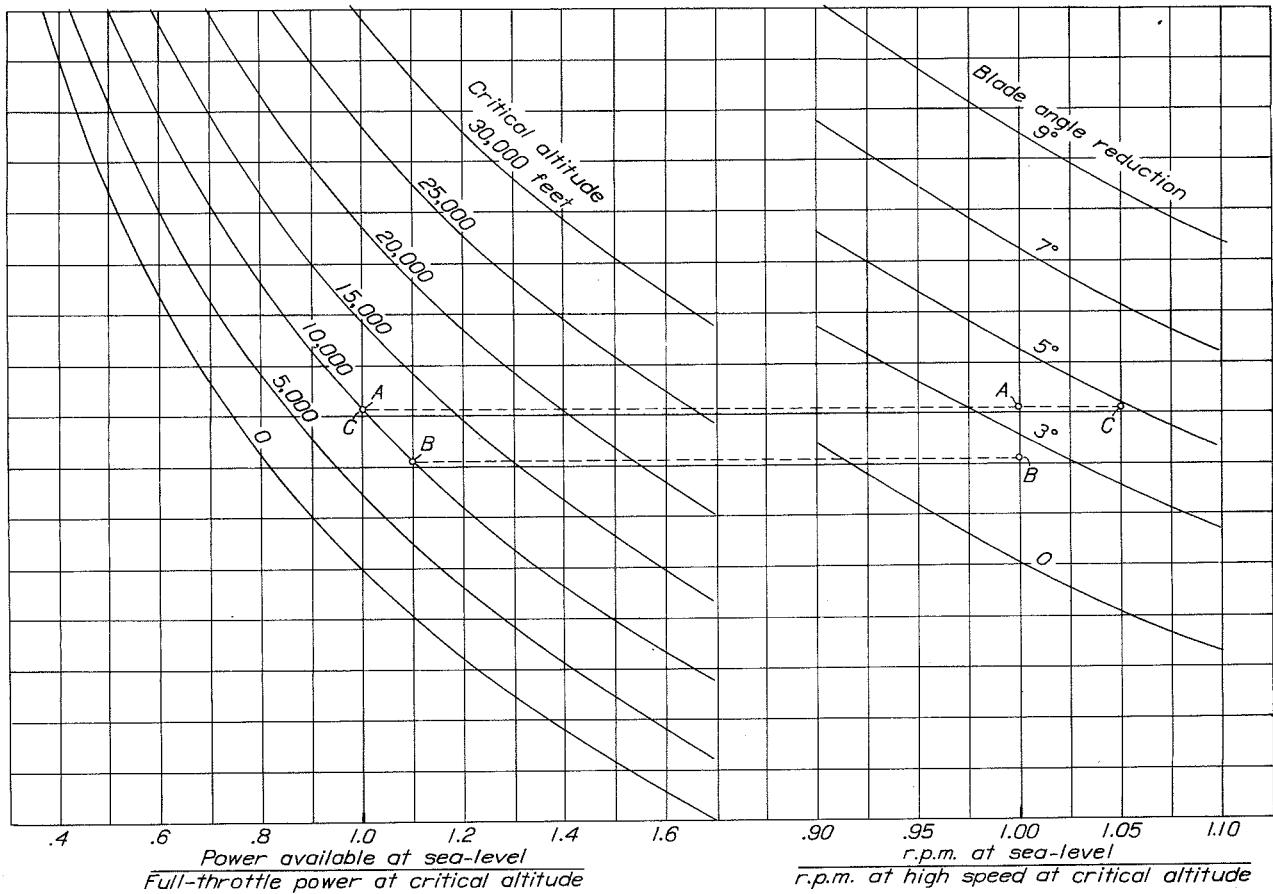


Figure 22a,b,c.- Comparison between various engine-propeller combinations for various basic pitch settings. Supercharged engine operating at various ratings at sea-level. (Critical altitude, 7,000 ft.)



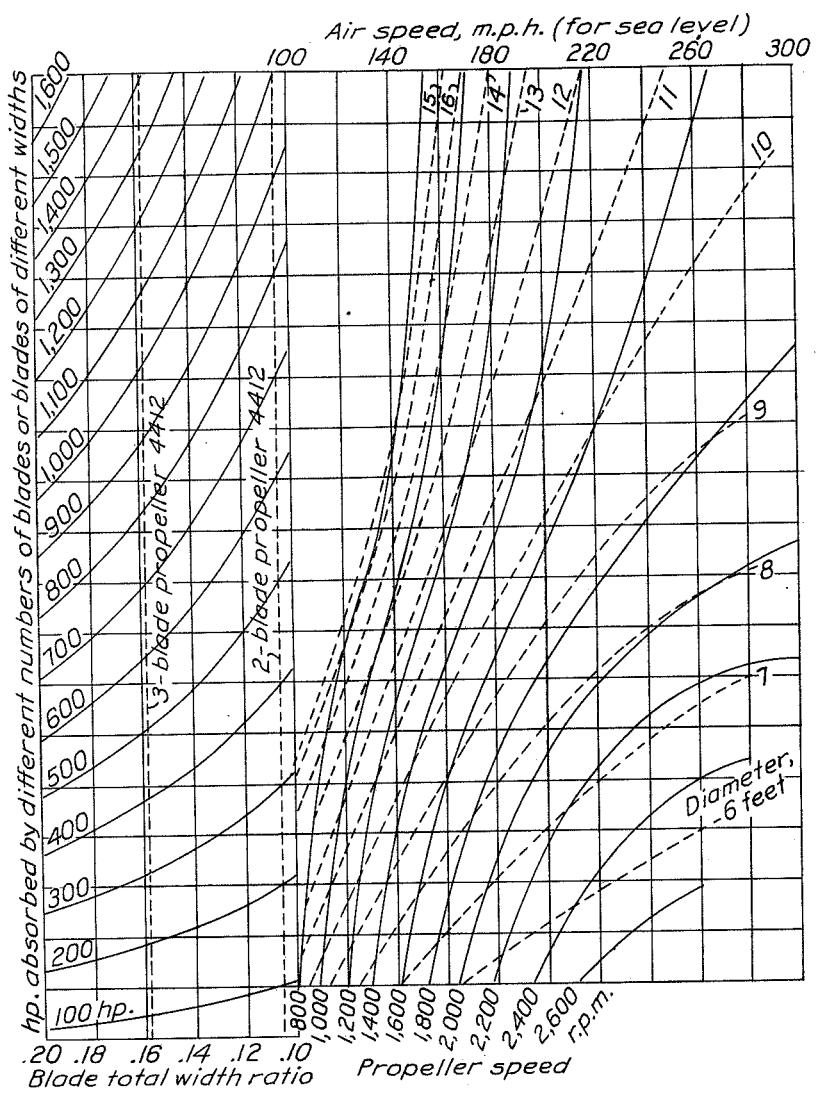


Figure 24.—Chart for obtaining diameter and r.p.m. for maximum high-speed efficiency. (A change of 400 r.p.m. will result in about 1 percent drop in efficiency.)

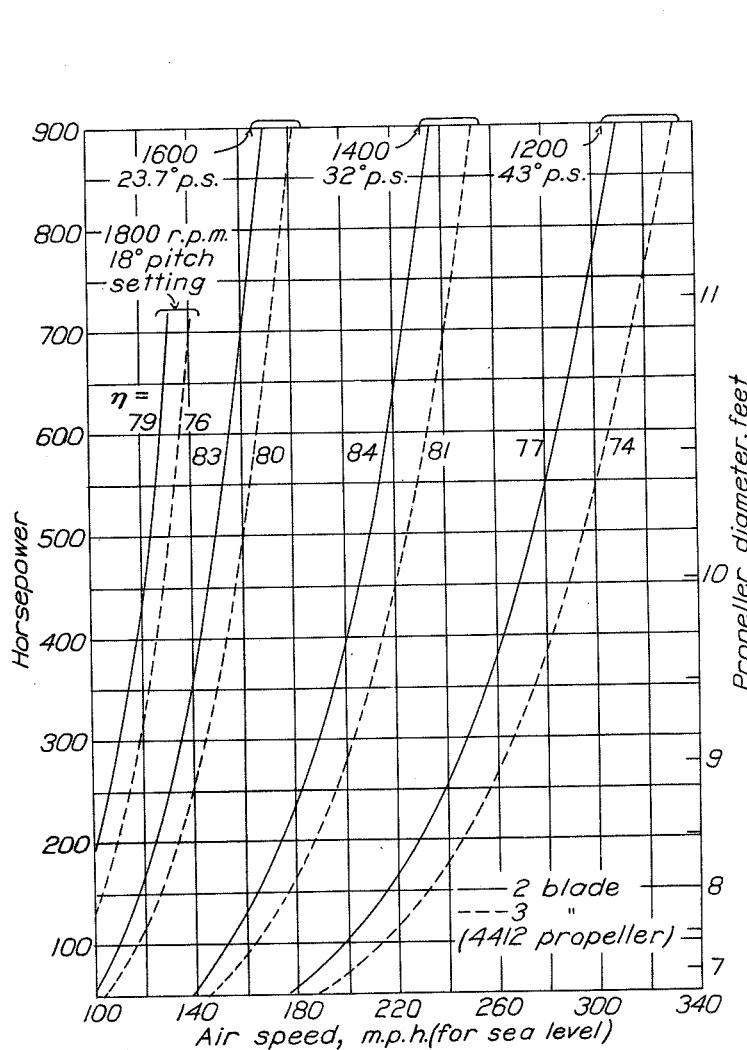
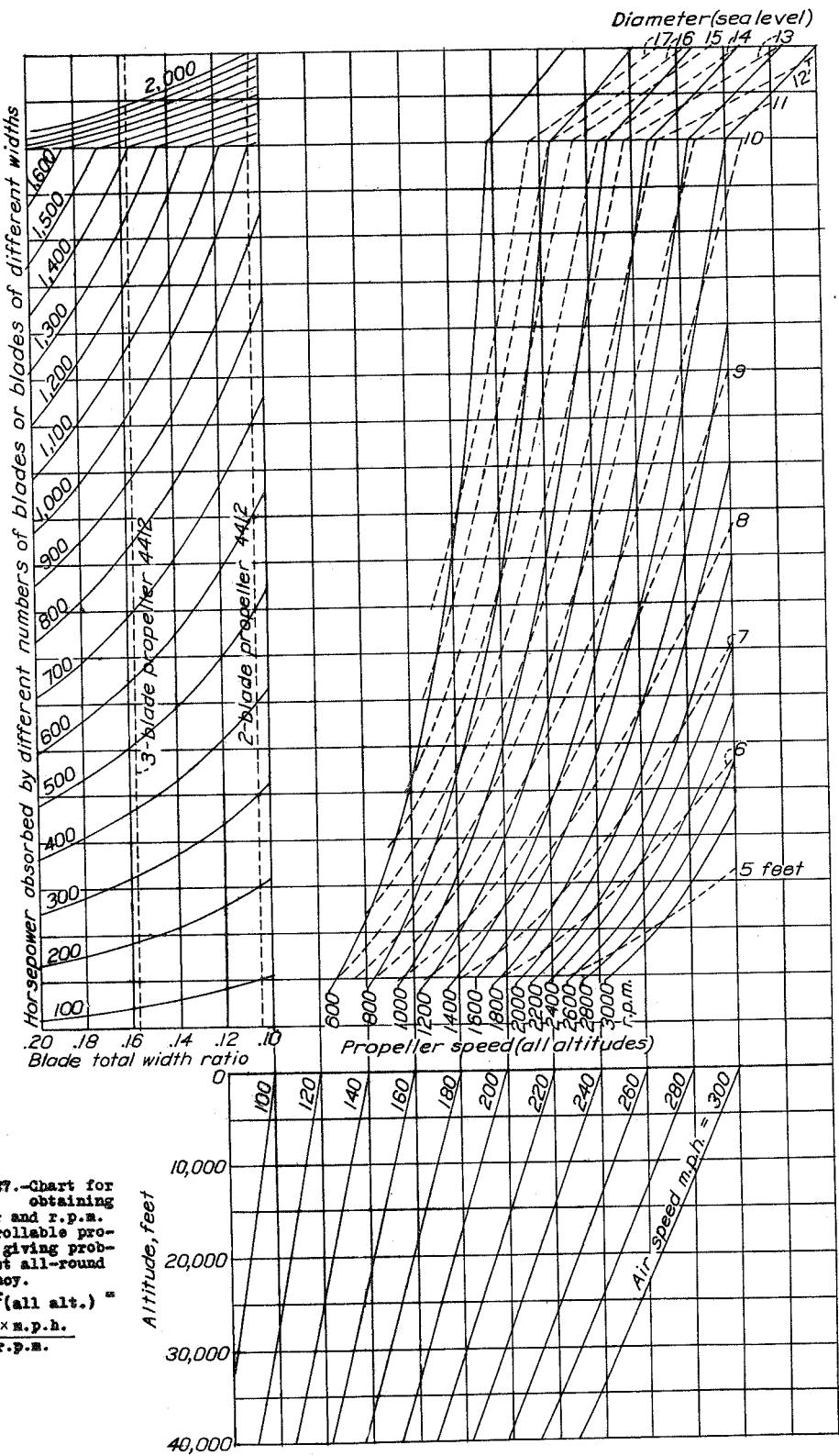


Figure 25.—Chart for obtaining best r.p.m. for conditions wherein propeller diameter is the limiting factor. Propeller diameter = 2.5 engine diameter (radial).  
2 blade --- 3 "  
(4412 propeller)



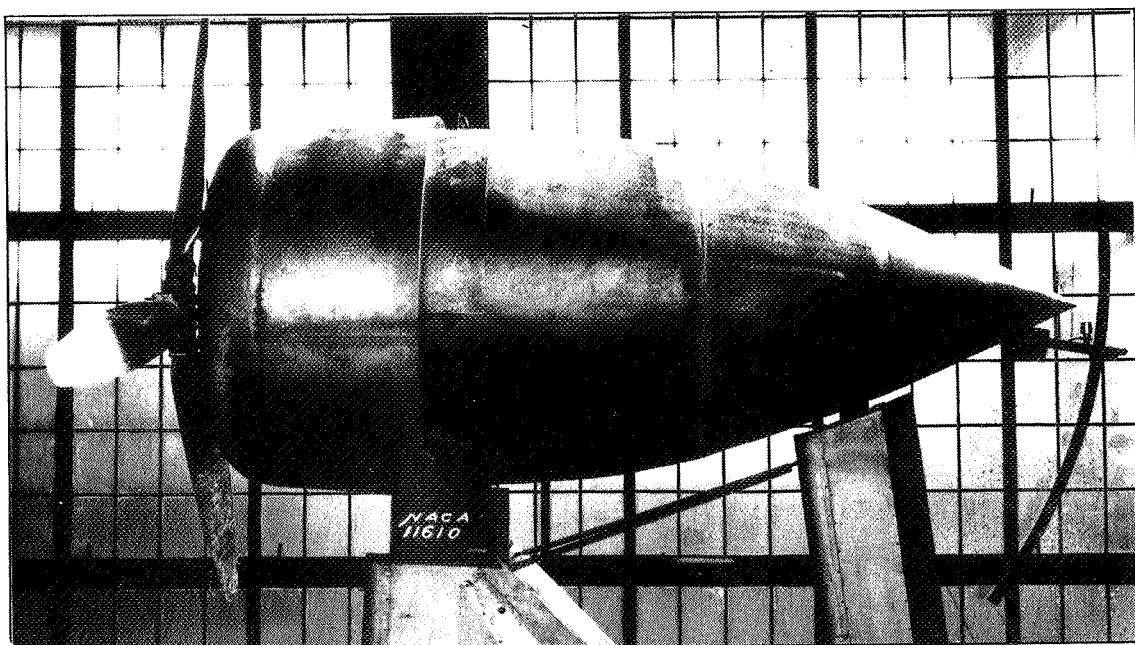


Figure 28.- Test nacelle.

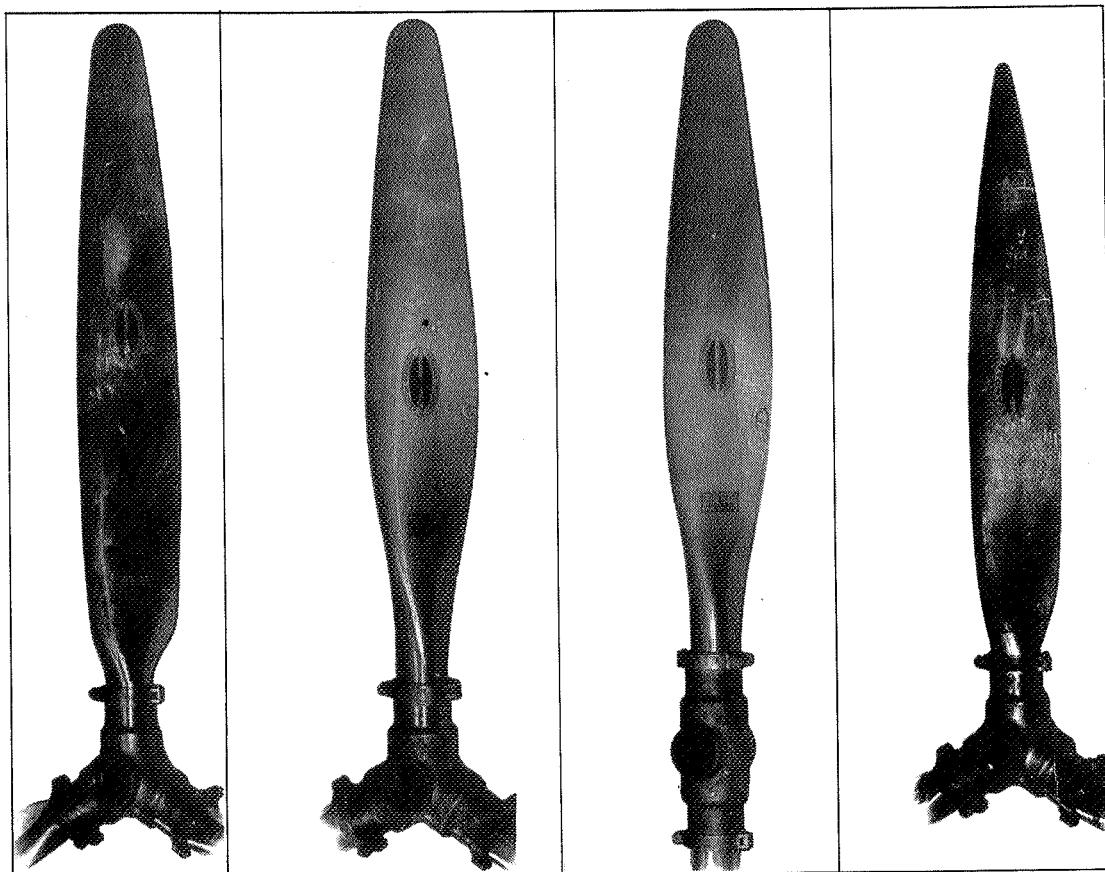
B, Drawing  
No. 101-0C, Drawing  
No. 5868-9D, Drawing  
No. 5868-9E, Drawing  
No. 3790

Figure 29.- Propellers B, C, D, and E.

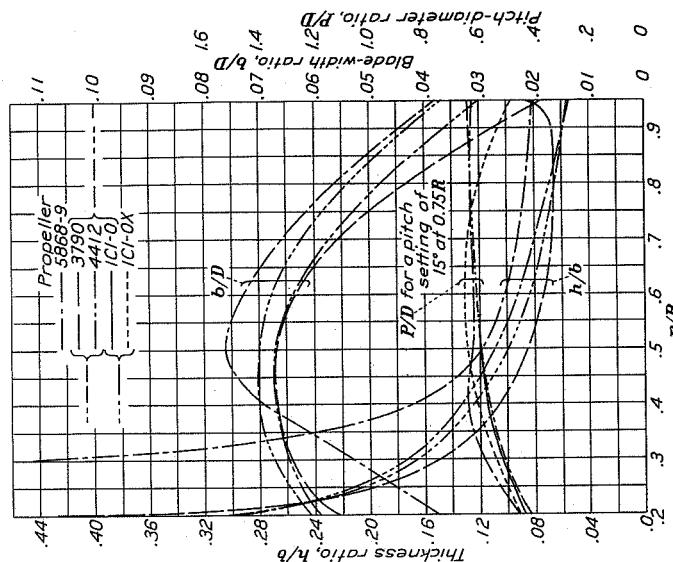


Figure 30. - Blade-form curves.

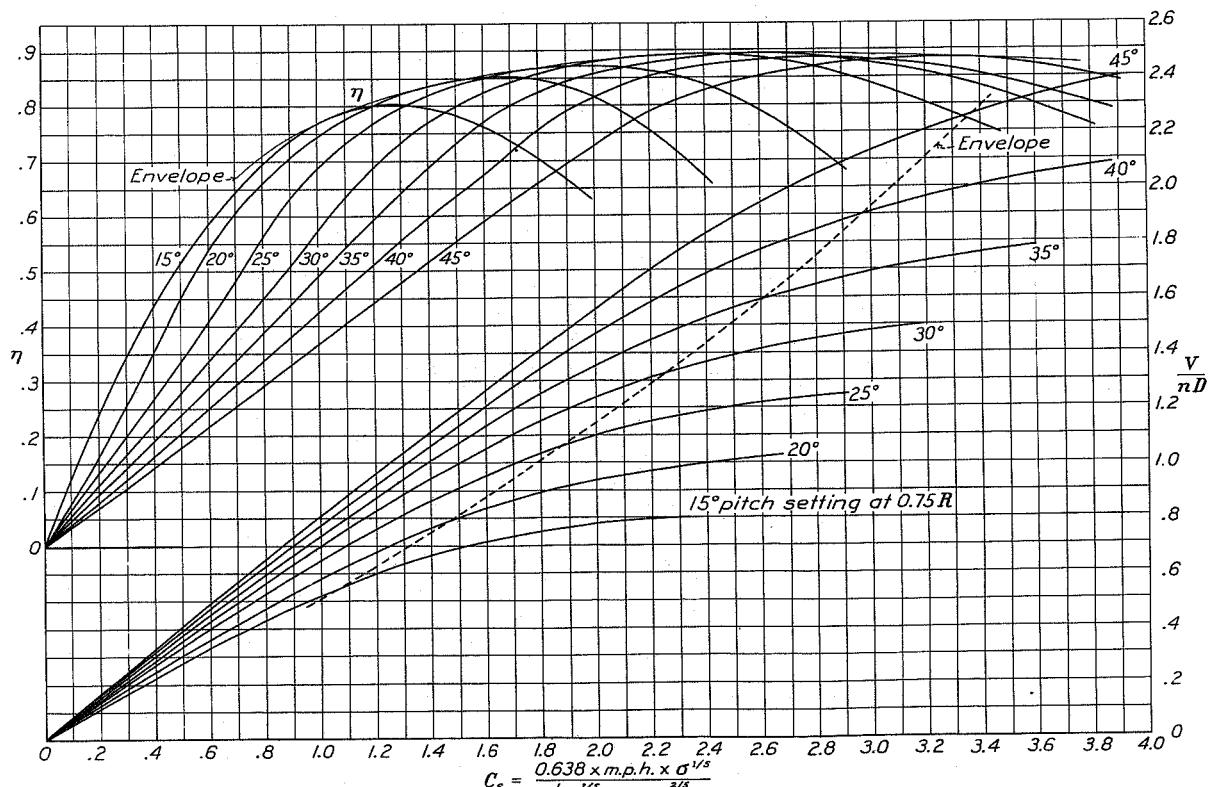


Figure 31. - Propeller B; Drawing No. 1C1-0; three blade; 10-foot diameter. N.A.C.A. cowled nacelle 4'-4" diameter.

N.A.C.A.

Figs. 32, 33

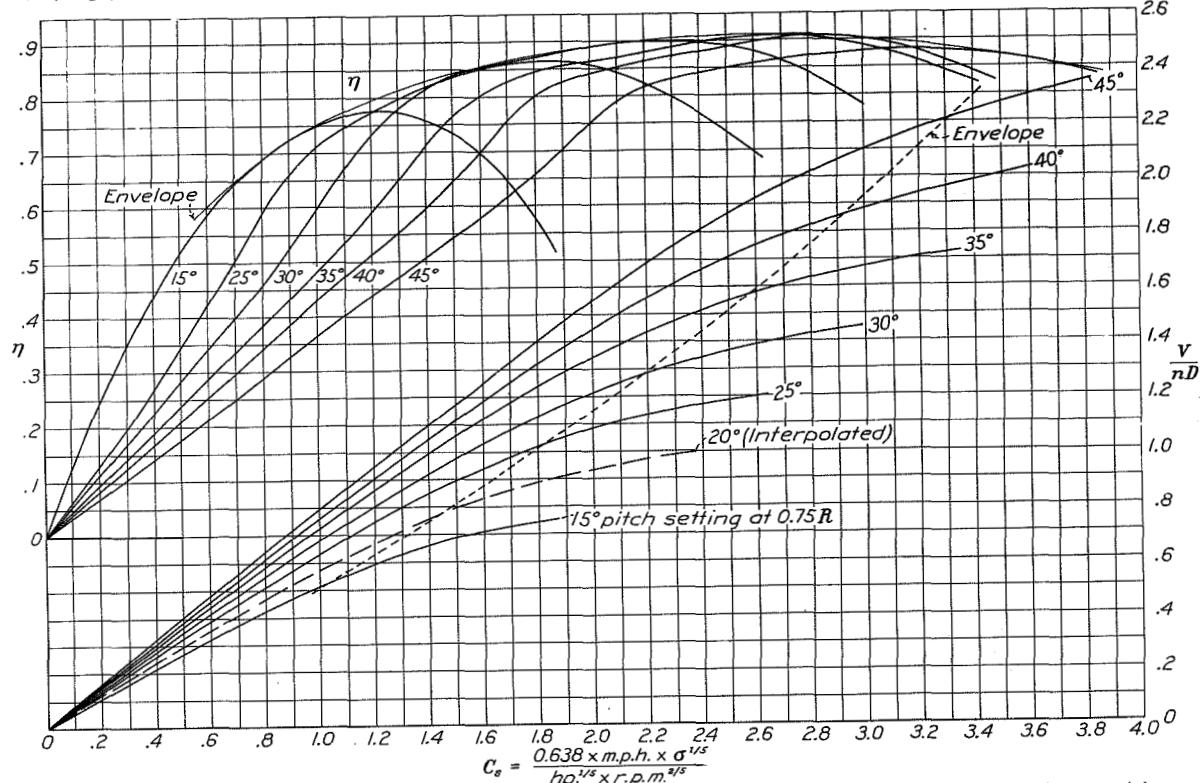


Figure 32.- Propeller B<sub>x</sub>; Drawing No. ICI-0X; special pitch distribution; three blade; 10-foot diameter. N.A.C.A. cowled nacelle 4'-4" diameter.

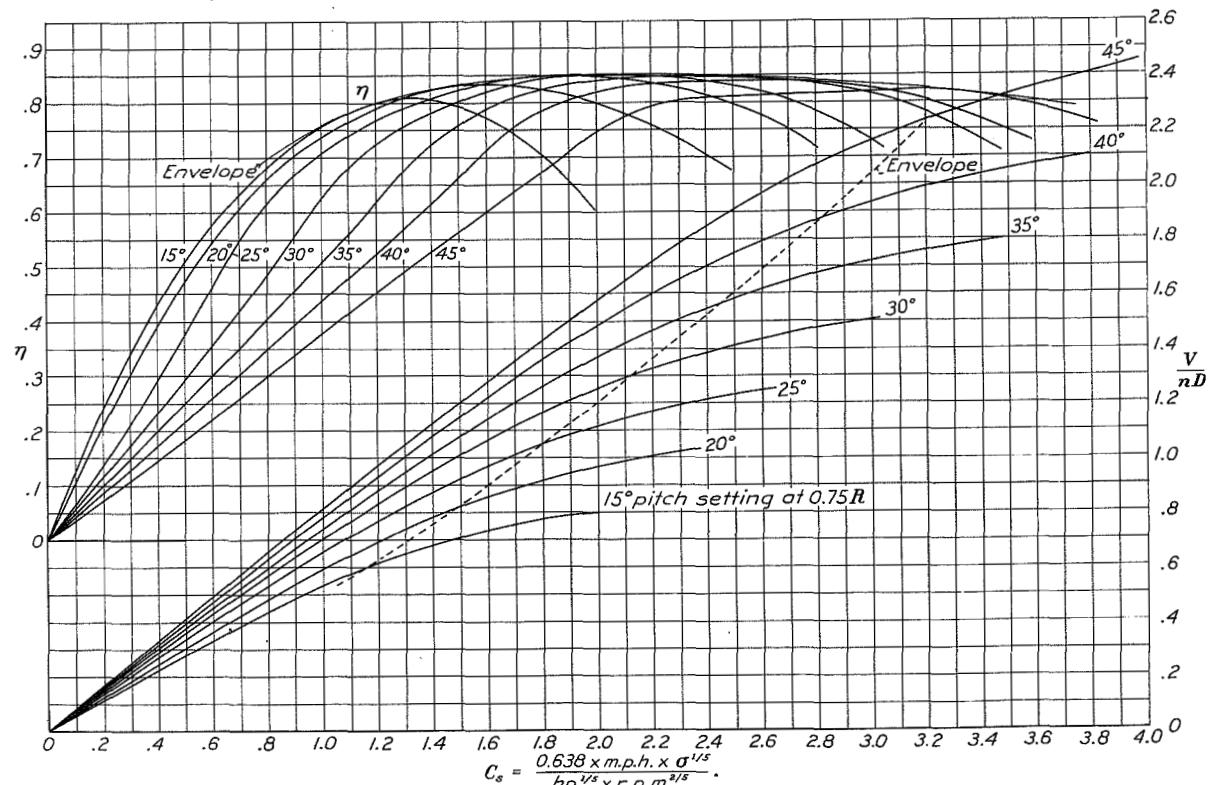


Figure 33.- Propeller C; Drawing No. 5868-9 three blade; 10-foot diameter. N.A.C.A. cowled nacelle 4'-4" diameter.

N.A.C.A.

Figs. 34, 35

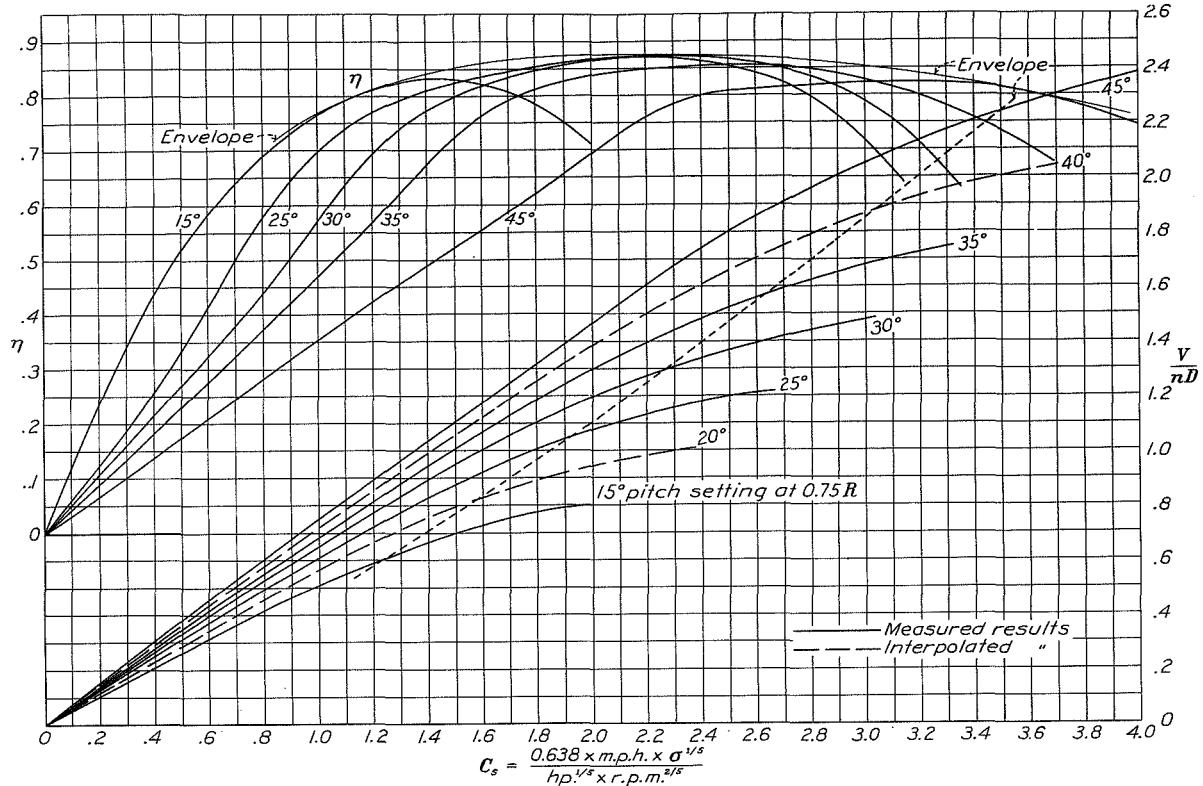


Figure 34.- Propeller D; Drawing No. 5868-S two blade; 10-foot diameter. N.A.C.A. cowled nacelle 4'-4" diameter.

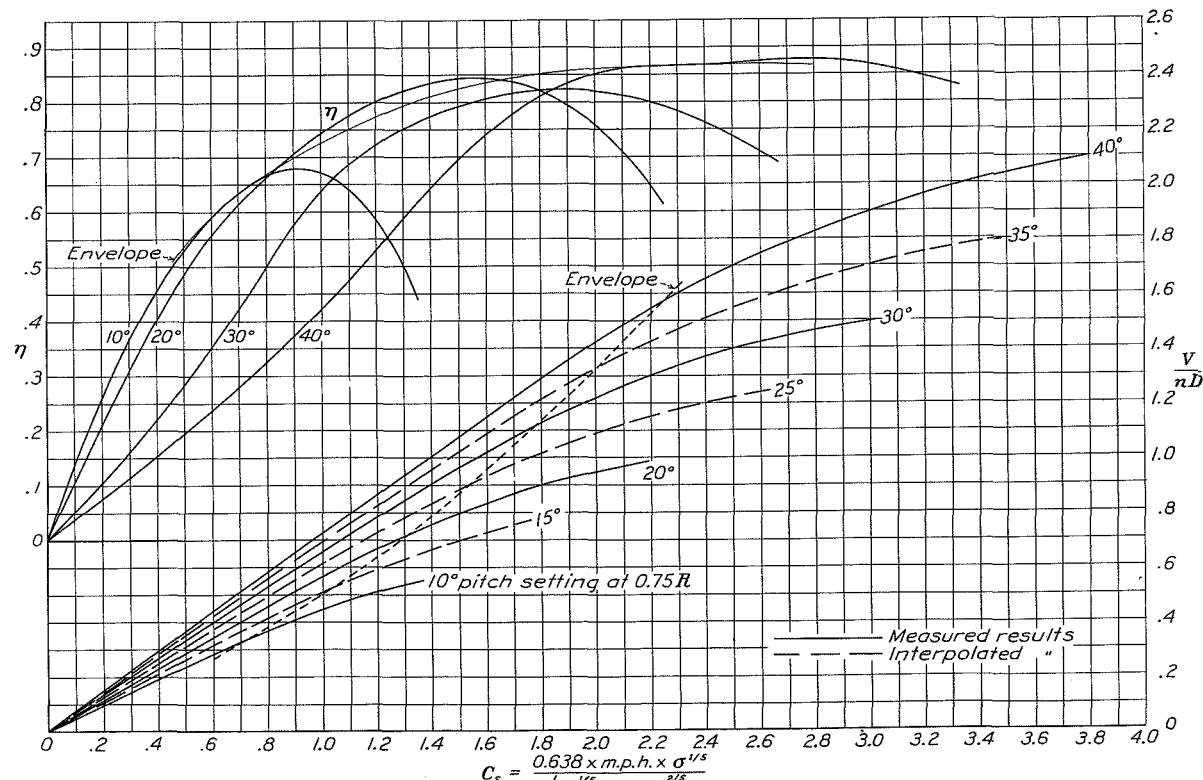


Figure 35.- Propeller E; Drawing No. 3790; three blade; 9-foot diameter. N.A.C.A. cowled nacelle 4'-4" diameter.

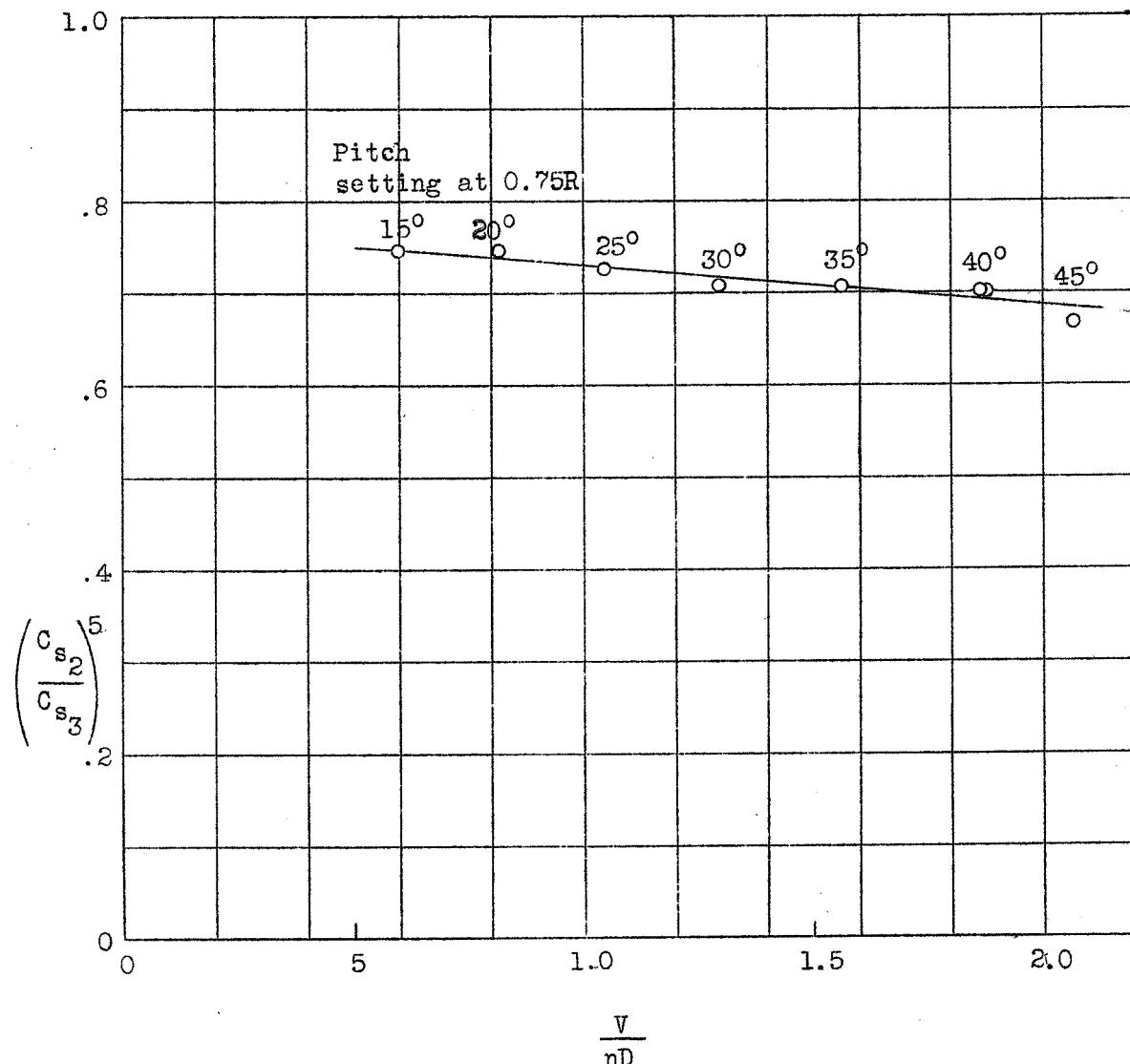


Figure 36.- Ratio of power absorbed by 2- and 3- blade propellers. Propellers 5868-9.

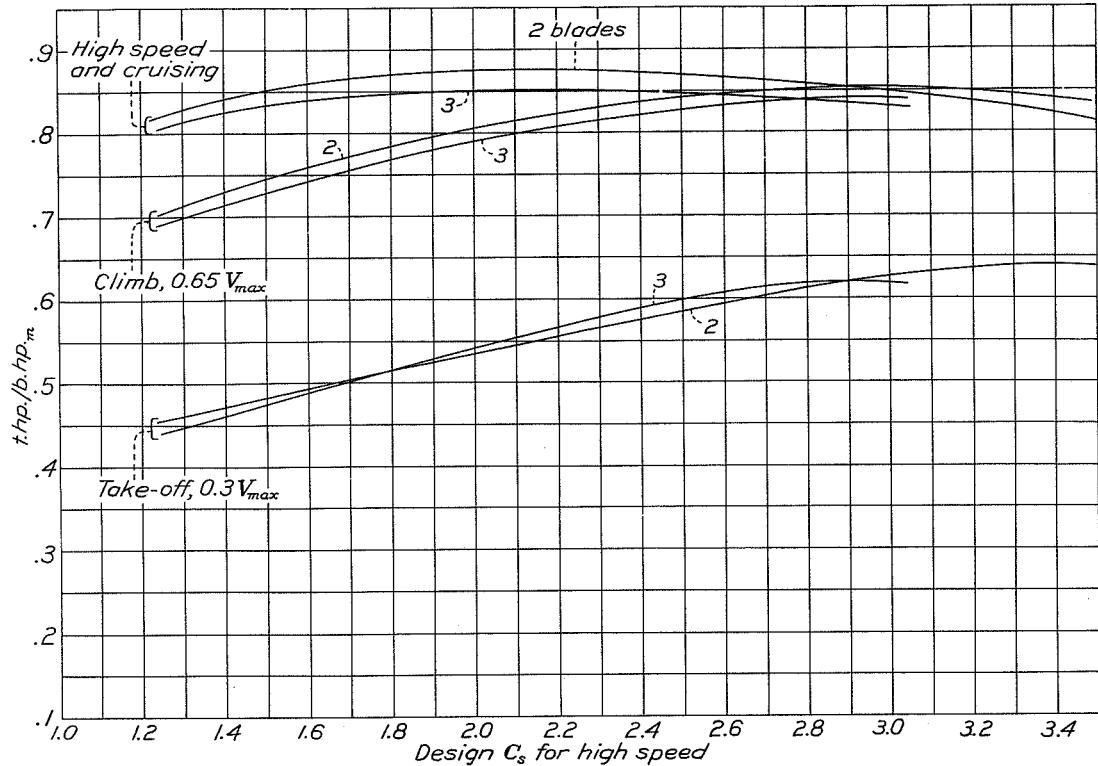


Figure 37. - Comparison between 2-and 3-blade propellers. Propeller 5868-9. Controllable propellers and unsupercharged engines.

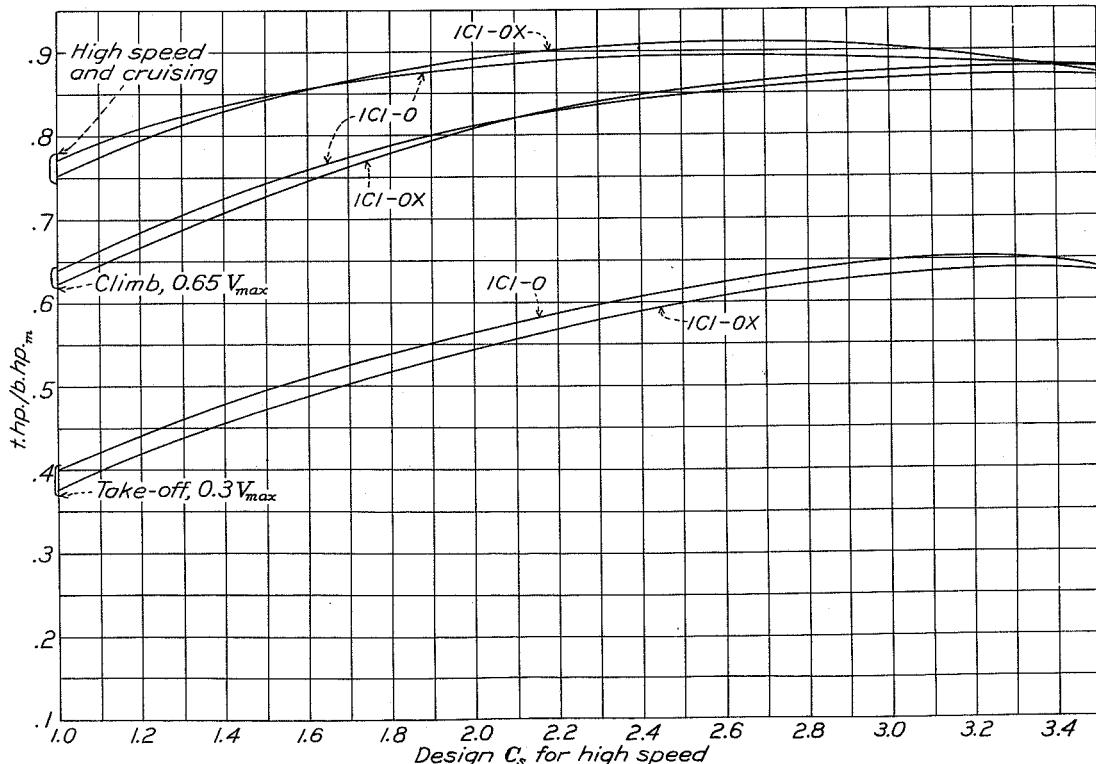
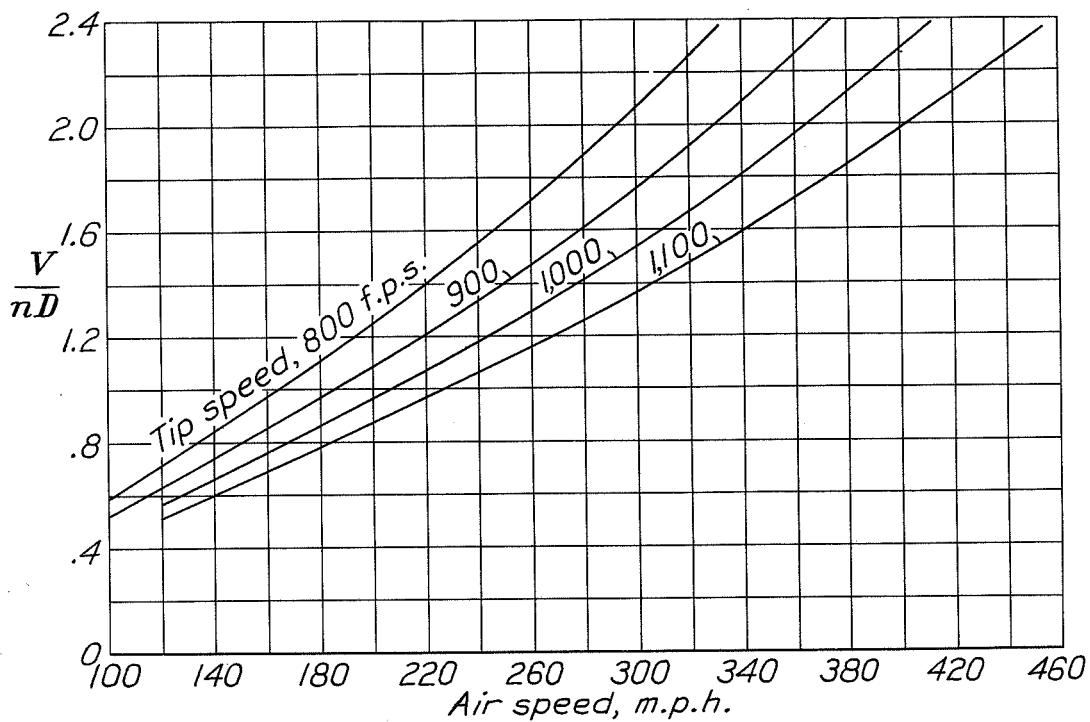
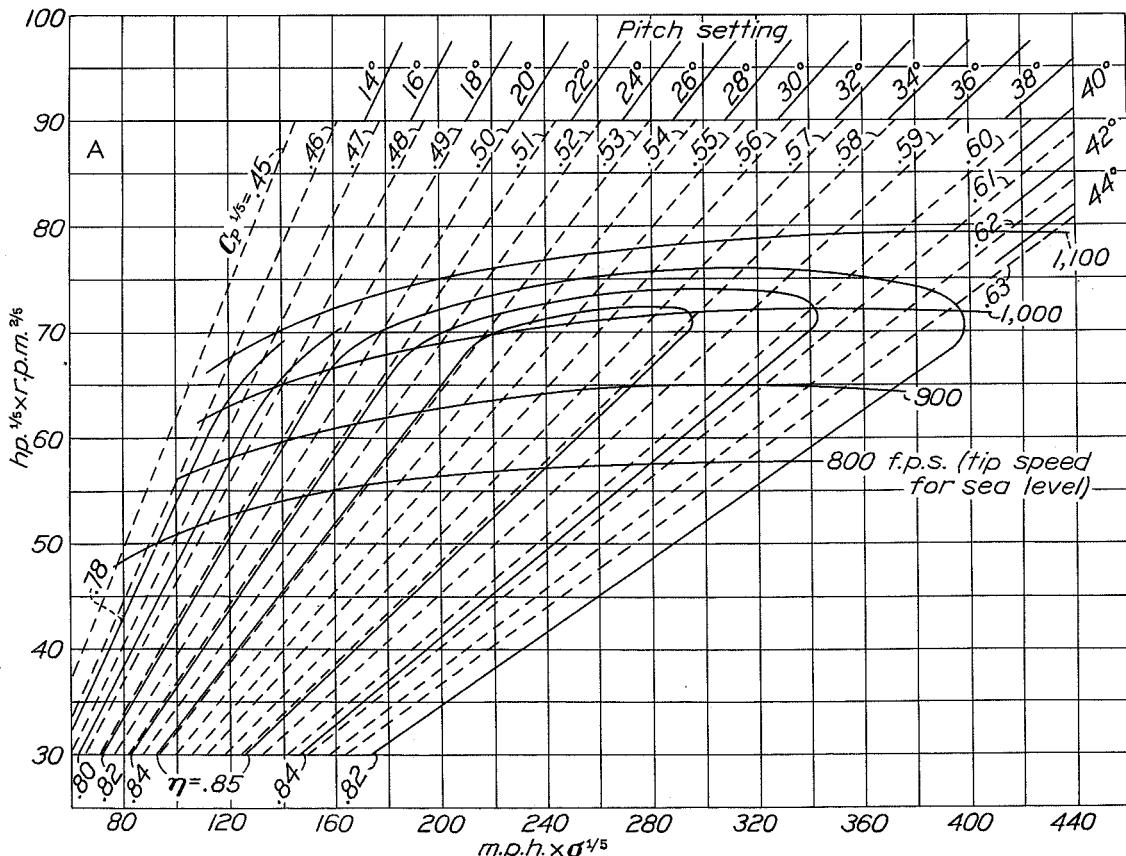


Figure 38. - Comparison between propellers having different pitch distributions. Controllable propellers and unsupercharged engines.

Figure 39.- Tip speeds for different air speeds and values of  $V/nD$ .Figure 40.- Maximum  $\eta$  at high speed. Propeller 5868-9, three blades. Total blade-width ratio, 0.1845.  
(To be used in conjunction with figure 2 only.)